

Corona Discharges: Modeling and Applications in Ionic Propulsions

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European
Innovation
Council



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the European Union

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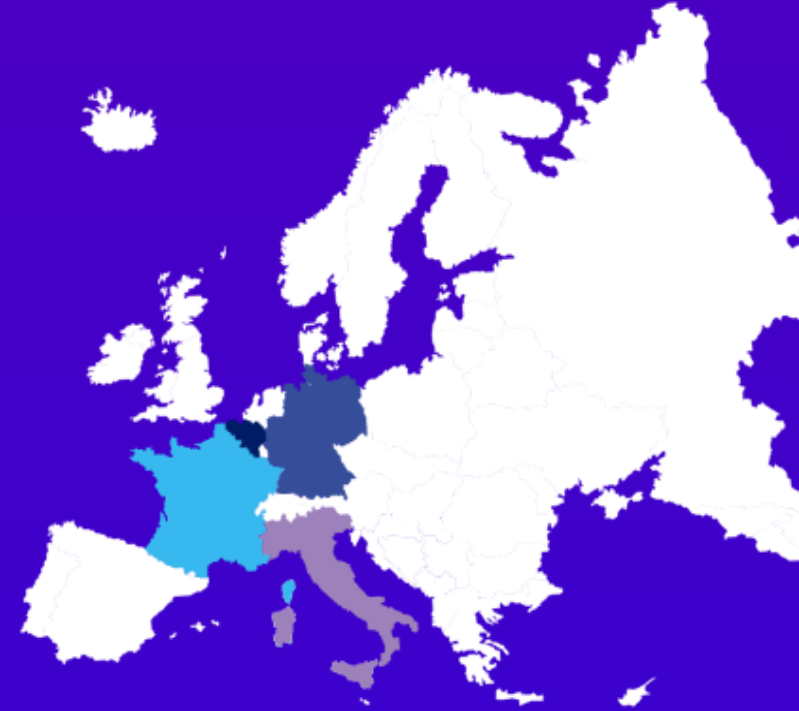


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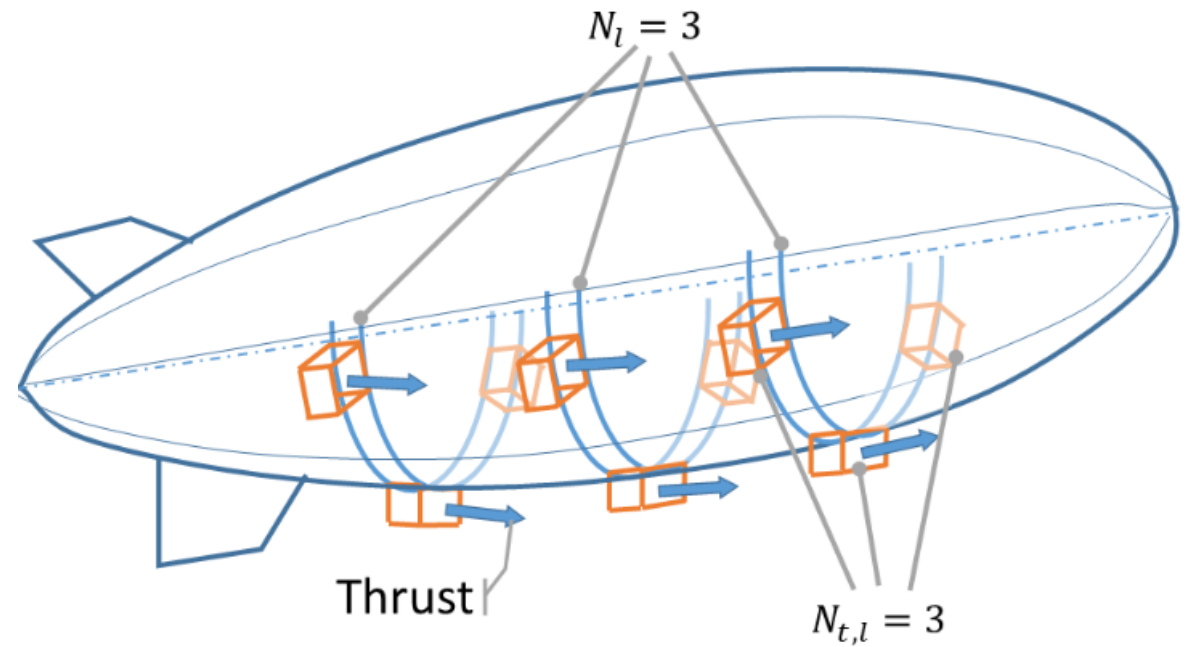
The IPROP Project

The Network of Partners

- POLIMI, Politecnico di Milano. Milano, Italy.
- UNIBO, Alma Mater Studiorum – Università di Bologna. Bologna, Italy.
- KIT, Karlsruher Institut fuer Technologie. Karlsruhe, Germany.
- VKI, Von Karman Institute for Fluid Dynamics. Sint-Genesius-Rode, Belgium.
- ISAE Supaero, Institut Supérieur de l'Aéronautique et de l'Espace. Toulouse, France.
- CNRS, Centre National de la Recherche Scientifique CNRS. Paris, France.
- AERONORD, Aeronord di Enzo Cisaro & C. s.a.s. Milano, Italy.
- TUD, Technische Universität Dresden. Dresden, Germany.



The IPROP project



Modeling Ionic Thrusters

$$\frac{\partial n_k}{\partial t} + \nabla \cdot (-D_k \nabla n_k + (z_k \mu_k \mathbf{E} + \mathbf{u}) n_k) = S_k \quad k = 1, \dots, N_{species}$$
$$-\nabla \cdot (\varepsilon \nabla \varphi) = \sum_{k=1}^{N_{species}} q_k n_k$$

$$\rho_f \frac{\partial \mathbf{u}}{\partial t} - \mu_f \Delta \mathbf{u} + \rho_f (\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla p = \mathbf{f}_{EHD}$$
$$\nabla \cdot \mathbf{u} = 0$$
$$\mathbf{f}_{EHD} = \sum_{k=1}^{N_{species}} q_k n_k \mathbf{E}$$

Corona Discharge
Responsible of thrust
generation



Fluid Problem
The airflow influences the
discharge

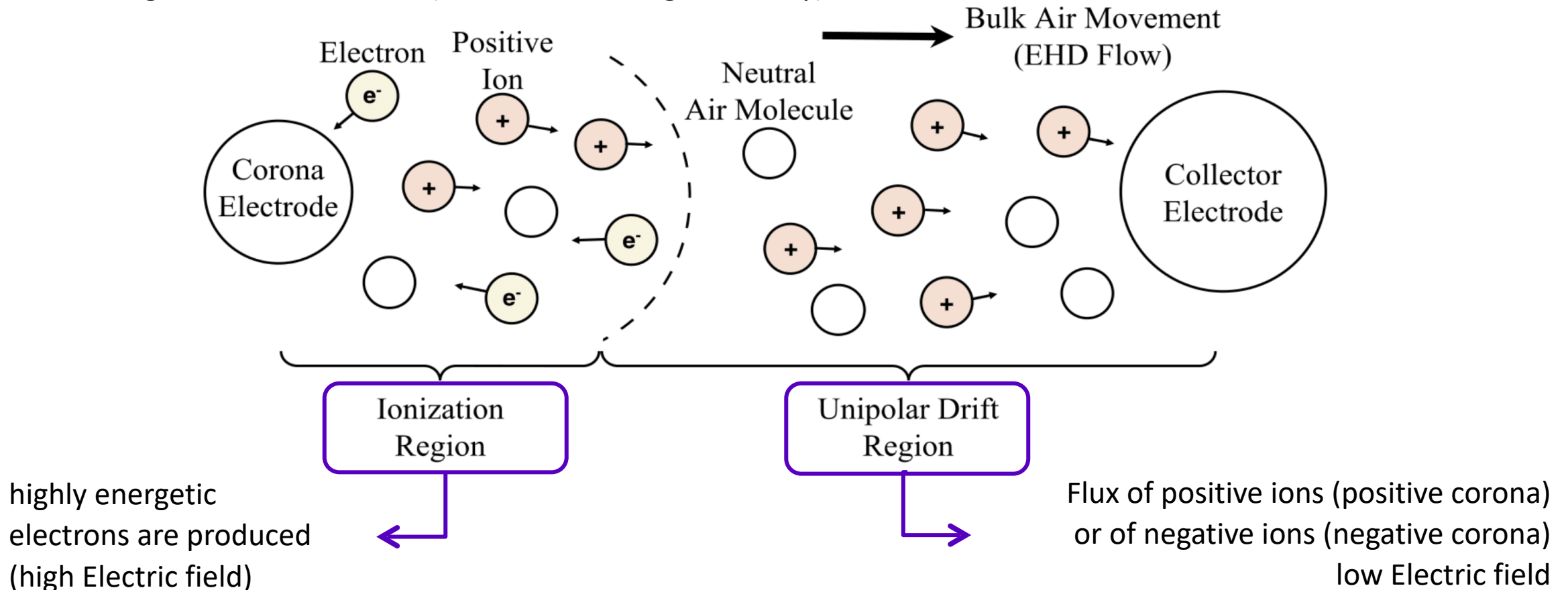
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Corona Discharges

Corona discharge

The corona discharge is a low/mid-current discharge caused by local breakdown of a gas gap with strongly inhomogeneous electric field (due to electrode geometry).

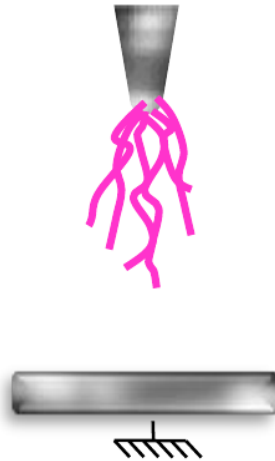


Phenomenology of corona discharges

Burst Corona



Streamer Corona



Glow Corona



Spark



Glow-Corona: steady current at a fixed voltage, quiet operation, and almost no sparking characterize this glow corona.

Streamer: the streamer regime is non-steady, quite audio noisy and emits strong radio noise.



Phenomenology of corona discharges

Non uniform electric field plays a key role in the formation of special properties of coronas

1. Minor contribution of ionization processes in the drift region of the corona, thus the **inter-electrode gap** is filled mainly with **negative or positive space charge**. This implies that the corona discharge is space-charge limited in magnitude, and that the volt-ampere characteristic (VAC) of the corona has a positive slope: an increase in current requires higher voltage to drive it.
2. Clear **distinction** between **drift** e **ionization region**
3. Townsend-like Discharge: electron avalanches play a key role. It is widely believed that **self-sustaining** of a positive corona is provided exclusively due to **photo-ionization** of the background gas.



3

Modeling Corona Discharges

Full-Scale Self-Consistent Model

Governing Equations

$$\frac{\partial n_k}{\partial t} + \nabla \cdot \Gamma_k = S_k \quad k = 1, \dots, N_{species}$$

$$-\nabla \cdot (\epsilon \nabla \varphi) = \sum_{k=1}^{N_{species}} q_k n_k$$

Assumptions

- Local Field Approximation (LFA)
Use a Boltzmann Solver to compute $T_E = f(E)$
- Fluxes through Drift-Diffusion Approximation
$$\Gamma_k = -D_k \nabla n_k + z_k \mu_k E n_k$$

Source Term

Air treated as a mixture of 80% N₂- 20% O₂

$$S_k = S_{chem,k} + S_{ph,k}$$

$$S_{ph}(r) = G(r) \int_{\Omega} \gamma_{ph} S_e(r') d^2 r'$$

BCs: Two Stream Approach

$$\Gamma_{ions} \cdot \hat{n} = \frac{1-r}{1+r} \left| \mathbf{v} \cdot \hat{n} \right| n_{ions}^{v_k = \text{sign}(q_k) \mu_k E n_k}$$

$$\Gamma_e \cdot \hat{n} = \frac{1-r_e}{1+r_e} \left| \mathbf{v} \cdot \hat{n} \right| n_e - \frac{2}{1+r_e} \sum_{ions} \gamma_{ions} \Gamma_{ions} \cdot \hat{n}$$

V. Gorin, A. Kudryavtsev, J. Yao, C. Yuan, Z. Zhou, Boundary conditions for drift-diffusion equations in gas-discharge plasma, Physics of Plasmas 27 (1) (2020) 013505



6-species model

- 13 reactions
- Rate Coefficients depend on the local electric field, gas temperature and electron temperature

No.	Reaction	Rate Coefficient
1a	$e^- + N_2 \rightarrow N_2^+ + e^- + e^-$	$\exp(-0.0105809 \cdot \ln^2 E^* - 2.40411 \cdot 10^{-75} \cdot \ln^{46} E^*) \text{ cm}^3/\text{s}$
1b	$e^- + O_2 \rightarrow O_2^+ + e^- + e^-$	$\exp(-0.0102785 \cdot \ln^2 E^* - 2.42260 \cdot 10^{-75} \cdot \ln^{46} E^*) \text{ cm}^3/\text{s}$
2a	$e^- + O_2^+ \rightarrow O_2$	$2.0 \cdot 10^{-7} \cdot (300/T_e)^{0.7} \text{ cm}^3/\text{s}$
2b	$e^- + N_2^+ \rightarrow N_2$	$2.8 \cdot 10^{-7} \cdot (300/T_e)^{0.5} \text{ cm}^3/\text{s}$
3a	$O_2^- + N_2^+ \rightarrow O_2 + N_2$	$2.0 \cdot 10^{-7} \cdot (300/T)^{0.5} \text{ cm}^3/\text{s}$
3b	$O_2^- + O_2^+ \rightarrow O_2 + O_2$	$2.0 \cdot 10^{-7} \cdot (300/T)^{0.5} \text{ cm}^3/\text{s}$
4a	$O_2^- + N_2^+ + N_2 \rightarrow O_2 + N_2 + N_2$	$2.0 \cdot 10^{-25} \cdot (300/T)^{2.5} \text{ cm}^6/\text{s}$
4b	$O_2^- + O_2^+ + N_2 \rightarrow O_2 + O_2 + N_2$	$2.0 \cdot 10^{-25} \cdot (300/T)^{2.5} \text{ cm}^6/\text{s}$
4c	$O_2^- + N_2^+ + O_2 \rightarrow O_2 + N_2 + O_2$	$2.0 \cdot 10^{-25} \cdot (300/T)^{2.5} \text{ cm}^6/\text{s}$
4d	$O_2^- + O_2^+ + O_2 \rightarrow O_2 + O_2 + O_2$	$2.0 \cdot 10^{-25} \cdot (300/T)^{2.5} \text{ cm}^6/\text{s}$
5a	$e^- + O_2 + O_2 \rightarrow O_2^- + O_2$	$ 1.4 \cdot 10^{-29} \cdot (300/T_e) \cdot \exp(-600/T) \cdot \exp\left(\frac{700(T_e-T)}{T_e T}\right) \text{ cm}^6/\text{s}$
5b	$e^- + O_2 + N_2 \rightarrow O_2^- + N_2$	$1.07 \cdot 10^{-31} \cdot (300/T_e)^2 \cdot \exp(-70/T) \cdot \exp\left(\frac{1500(T_e-T)}{T_e T}\right) \text{ cm}^6/\text{s}$
6	$O_2^- + O_2 \rightarrow e^- + O_2 + O_2$	$8.6 \cdot 10^{-10} \cdot \exp(-6030/T) \cdot (1 - \exp(-1570/T)) \text{ cm}^3/\text{s}$

B. Parent, S.O. Macheret, and M.N. Shneider. Electron and ion transport equations in computational weakly-ionized plasmadynamics. Journal of Computational Physics, 259:51–69, 201.





3D Photoionization Source Term

Definition

$$S_{ph}(\vec{R}) = \int_{V'} \frac{I(\vec{R}')g(\mathcal{R})}{4\pi\mathcal{R}^2} dV'$$

$$\mathcal{R} = |\vec{R} - \vec{R}'|$$

3D Photoionization Kernel

$$\frac{g(\mathcal{R})}{p_{O_2}} = \frac{e^{-\lambda_1\mathcal{R}} - e^{-\lambda_2\mathcal{R}}}{p_{O_2}\mathcal{R} \ln \frac{\lambda_2}{\lambda_1}}$$

Ionization Source term

$$I(\vec{R}) = \gamma_{ph} S^{1a}(\vec{R})$$

$$\gamma_{ph} = \left(0.03 + \frac{15.7 Td}{E/N_{gas}} \right) \frac{p_q}{p_{gas} + p_q}$$

M.B. Zheleznyak, A.K. Mnatsakanyan, S.V. Sizykh, Photoionization of nitrogen and oxygen mixtures by radiation from a gas discharge, High Temp. 93 (20) (1982) 357–362

G. Calìò, F. Ragazzi, A. Popoli, A. Cristofolini, L. Valdetaro, C. de Falco, and P. Barbante. Hierarchical multiscale modeling of positive corona discharges. J. Electrostatics, 141:104291, 2026

M. S. Benilov, P.G.C. Almeida, N. G. C. Ferreira, R. M. S. Almeida, and G. V. Naidis. A practical guide to modeling low-current quasi-stationary gas discharges: Eigenvalue, stationary, and time-dependent solvers. Journal of Applied Physics, 130(12):121101, 2021





2D Photoionization Source Term

$$\vec{R} = \vec{r} + \vec{z} \quad \text{Decomposition of the position vector}$$

The general three-dimensional kernel can be integrated along the longitudinal axis

$$G(\rho) = \frac{1}{4\pi \ln(\frac{\lambda_2}{\lambda_1})} \int_{-\infty}^{\infty} \frac{e^{-\lambda_1 \sqrt{\rho^2 + z^2}} - e^{-\lambda_2 \sqrt{\rho^2 + z^2}}}{(\rho^2 + z^2)^{3/2}} dz \quad \rho = |\vec{r} - \vec{r}'|$$

2D Photoionization Kernel

$$G(\rho) = \frac{\rho}{32\pi \ln(\frac{\lambda_2}{\lambda_1})} \left(\lambda_1^3 G_{1,3}^{3,0} \left(\frac{\lambda_1^2 \rho^2}{4} \middle| -\frac{1}{2}, -1, -\frac{3}{2} \right) - \lambda_2^3 G_{1,3}^{3,0} \left(\frac{\lambda_2^2 \rho^2}{4} \middle| -\frac{1}{2}, -1, -\frac{3}{2} \right) \right)$$

N. Monrolin and F. Plouraboué. Multi-scale two-domain numerical modeling of station-ary positive DC corona discharge/drift-region coupling . J. Comp. Phys, 443:110517,2021





Asymptotic Treatment of S_{ph}

$$S_{ph}(\vec{R}) = \int_{V'} \frac{I(\vec{R}')g(\mathcal{R})}{4\pi\mathcal{R}^2} dV' \longrightarrow S_{ph}(\vec{r}) = \int_{\Omega} \gamma_{ph} S^{1a}(\vec{r}') G(\rho) d^2\vec{r}'$$

The kernel strongly decreases as a function of its argument and it is convenient to expand it inside the integral

$$S_{ph}(\vec{r}) = G(|\vec{r}|) \int_{\Omega} \gamma_{ph} S^{1a}(\vec{r}') d^2\vec{r}' + \nabla G(|\vec{r}|) \cdot \int_{\Omega} \gamma_{ph} S^{1a}(\vec{r}') \vec{r}' d^2\vec{r}' + O(\rho^2)$$

$$S_{ph}(\vec{r}) = S_{ph,0}(\vec{r}) + S_{ph,1}(\vec{r}) + O(\rho^2)$$

$$S_{ph}(\vec{r}) \approx S_{ph,0}(\vec{r})$$

G. Calìò, F. Ragazzi, A. Popoli, A. Cristofolini, L. Valdetaro, C. de Falco, and P. Barbante. Hierarchical multiscale modeling of positive corona discharges. J. Electrostatics, 141:104291, 2026

N. Monrolin and F. Plouraboué. Multi-scale two-domain numerical modeling of station-ary positive DC corona discharge/drift-region coupling . J. Comp. Phys, 443:110517, 2021





Self Consistent 3-species Model

$$\begin{aligned}
 -\nabla \cdot (\varepsilon \nabla \varphi) &= q(n_p - n_n - n_e) \\
 \nabla \cdot (-D_p \nabla n_p - \mu_p n_p \nabla \varphi) &= \alpha j_e + S_{ph} \\
 \nabla \cdot (-D_n \nabla n_n + \mu_n n_n \nabla \varphi) &= \eta j_e \\
 \nabla \cdot (-D_e \nabla n_e + \mu_e n_e \nabla \varphi) &= (\alpha - \eta) j_e + S_{ph} \\
 -\nabla \cdot (\nabla S_i) + (\lambda_i p_{O_2})^2 S_i &= \gamma_{ph} A_i p_{O_2}^2 \alpha j_e \quad \text{for } j = 1, 2, 3 \\
 j_e = \mu_e |E| n_e \quad S_{ph} &= \sum_{i=1}^3 S_i
 \end{aligned}$$

3-species model

- 2 reactions: impact ionization and attachment
- 3 terms Helmholtz model for the photionization term

Implementation

Jacobian computed from linearization of the residual through SACADO

A. Bourdon, V.P. Pasko, N.Y. Liu, S. Célestin, P. Ségur, and E. Marode. Efficient models for photoionization produced by non-thermal gas discharges in air based on radiative transfer and the helmholtz equations. Plasma Sources Science and Technology, 16(3): 656, 2007





$$\begin{aligned}
 -\nabla \cdot (\varepsilon \nabla \varphi) &= q(n_p - n_n - n_e) \\
 \nabla \cdot (-D_p \nabla n_p - \mu_p n_p \nabla \varphi) &= \alpha j_e + S_{ph} \\
 \nabla \cdot (-D_n \nabla n_n + \mu_n n_n \nabla \varphi) &= \eta j_e \\
 \nabla \cdot (-D_e \nabla n_e + \mu_e n_e \nabla \varphi) &= (\alpha - \eta) j_e + S_{ph} \\
 -\nabla \cdot (\nabla S_i) + (\lambda_i p_{O_2})^2 S_i &= \gamma_{ph} A_i p_{O_2}^2 \alpha j_e \quad \text{for } j = 1, 2, 3
 \end{aligned}$$

$$j_e = \mu_e |E| n_e$$

$$S_{ph} = \sum_{i=1}^3 S_i$$

$$\begin{aligned}
 \alpha &= N_{gas} B e^{-\frac{C}{E/N_{gas}}} \\
 \eta &= N_{gas} \frac{B_\eta e^{-\frac{C_\eta}{E/N_{gas}}}}{\left(\frac{E/N_{gas}}{C_\eta}\right)^{D_\eta}}
 \end{aligned}$$

N. Monrolin and F. Plouraboué. Multi-scale two-domain numerical modeling of station-ary positive DC corona discharge/drift-region coupling . J. Comp. Phys, 443:110517,2021

$$\gamma_{ph} = \left(0.03 + \frac{15.7 Td}{E/N_{gas}}\right) \frac{p_q}{p_{gas} + p_q}$$

M. S. Benilov, P.G.C. Almeida, N. G. C. Ferreira, R. M. S. Almeida, and G. V. Naidis. A practical guide to modeling low-current quasi-stationary gas discharges: Eigenvalue, stationary, and time-dependent solvers. Journal of Applied Physics, 130(12):121101, 2021





Macro-Scale Model (1)

Poisson + Drift-Diffusion eqs.

$$-\nabla \cdot (\varepsilon \nabla \varphi) = q n$$

$$-\nabla \cdot (D \nabla n + \mu \nabla \varphi n) = 0$$

Boundary Conditions

$$\begin{aligned} \varphi &= V_a \quad \text{on} \quad \partial\Omega_e; \\ -\nabla \varphi \cdot \hat{n} &= E_{ON} \quad \text{on} \quad \partial\Omega_e; \\ \varphi &= 0 \quad \text{on} \quad \partial\Omega_c; \\ \nabla \varphi \cdot \hat{n} &= 0 \quad \text{on} \quad \partial\Omega \setminus (\partial\Omega_e \cup \partial\Omega_c) \end{aligned}$$

$$J \cdot \hat{n} = 0 \quad \text{on} \quad \partial\Omega$$

Kaptzov Hypothesis

The electric field on the emitting electrode increases proportionally to the voltage until a certain value, called the corona onset level. After that the electrical field at the injecting electrode remains unchanged and equal to the onset value.

Onset electric field E_{ON} computed with Peek's law at $p=1$ atm:

$$E_{ON} = 3,1 \times 10^6 \left(1 + \frac{0,308}{\sqrt{R_e}} \right)$$

D. Coseru S., Fabre and F. Plouraboué. Numerical study of ElectroAeroDynamic force and current resulting from ionic wind in emitter/collector systems. J. App. Phys., 129(10):85–96, 2021





Macro-Scale Model (2)

Poisson + Drift-Diffusion eqs.

$$\begin{aligned} -\nabla \cdot (\varepsilon \nabla \varphi) &= q n \\ -\nabla \cdot (D \nabla n + \mu \nabla \varphi n) &= 0 \end{aligned}$$

Boundary Conditions

$$\begin{aligned} \varphi &= V_a \quad \text{on} \quad \partial\Omega_e; \\ \varphi &= 0 \quad \text{on} \quad \partial\Omega_c \end{aligned}$$

$$\nabla \varphi \cdot \hat{n} = 0 \quad \text{on} \quad \partial\Omega \setminus (\partial\Omega_e \cup \partial\Omega_c)$$

$$n = f(E \cdot \hat{n}) \quad \text{on} \quad \partial\Omega_e$$

$$n = n_{min} \quad \text{on} \quad \partial\Omega_c$$

$$J \cdot \hat{n} = 0 \quad \text{on} \quad \partial\Omega \setminus (\partial\Omega_e \cup \partial\Omega_c)$$

Kaptzov Hypothesis

The electric field on the emitting electrode increases proportionally to the voltage until a certain value, called the corona onset level. After that the electrical field at the injecting electrode remains unchanged and equal to the onset value.

To enforce approximately this behaviour by means of a local condition we let:

$$n = n_{ref} e^{\frac{E \cdot \hat{n} - E_{ON}}{E_{ref}}} \quad \text{on} \quad \partial\Omega_e$$

Corona inception when $|E| > E_{ON}$

G. Calìò, F. Ragazzi, A. Popoli, A. Cristofolini, L. Valdetaro, C. de Falco, and P. Barbante. Hierarchical multiscale modeling of positive corona discharges. J. Electrostatics, 141:104291, 2026



Results

Results

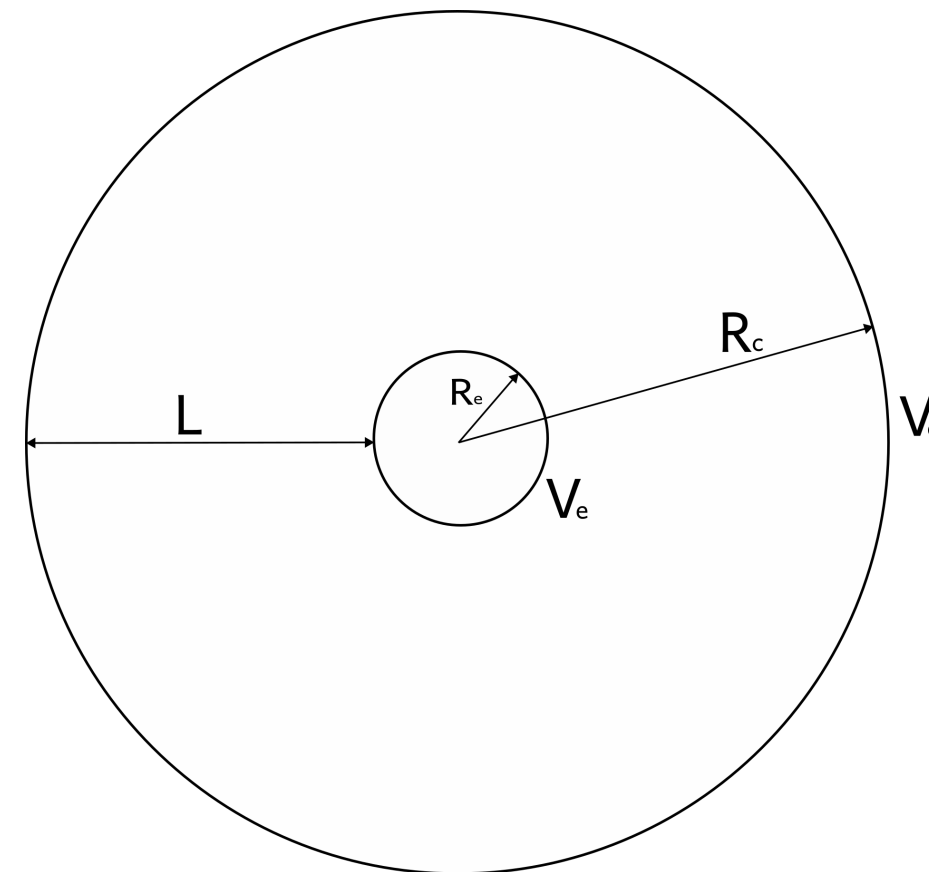


Geometry data:

	Value	Unit
Gap Length L	10.35	cm
Emitter Radius R_e	700	μm

Applied Voltage:

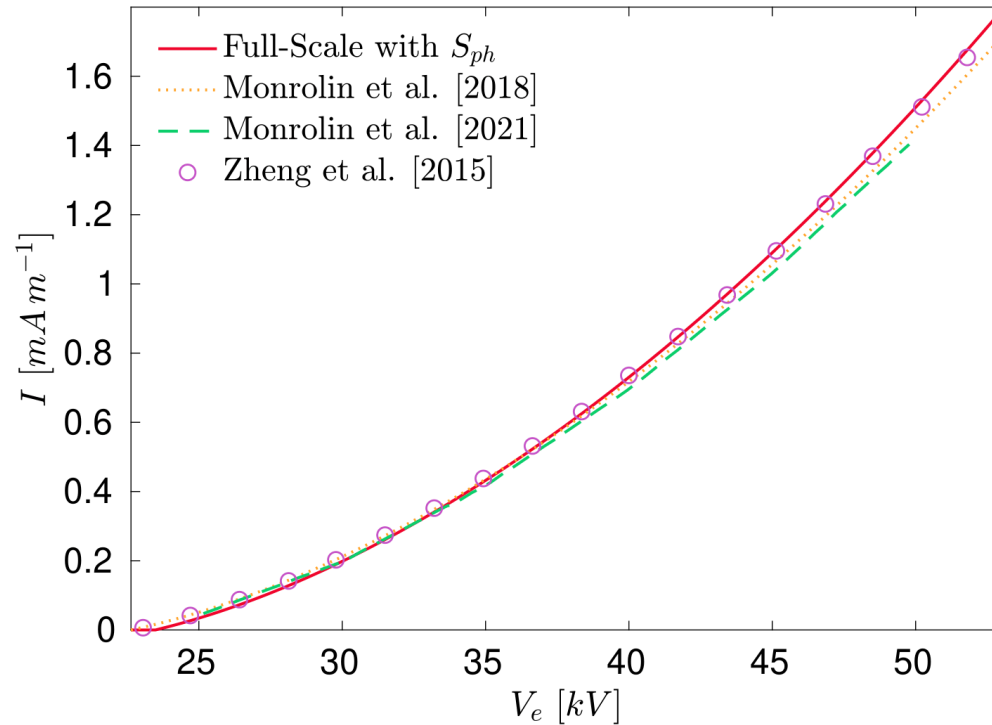
	Value	Unit
Collector Voltage	0	V
Emitter Voltage	From 5 up to 55	kV



Results: Full-Scale Model

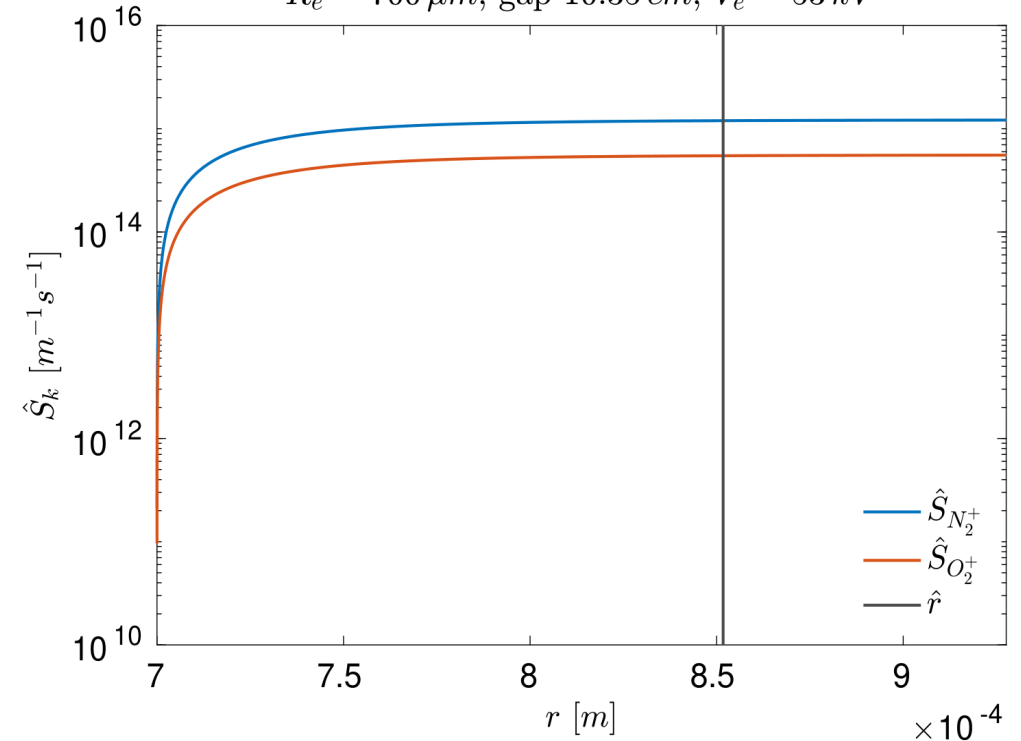
Current-voltage

$R_e = 700 \mu m$, gap $10.35 cm$



Cumulative integral of reaction rates

$R_e = 700 \mu m$, gap $10.35 cm$, $V_e = 53 kV$



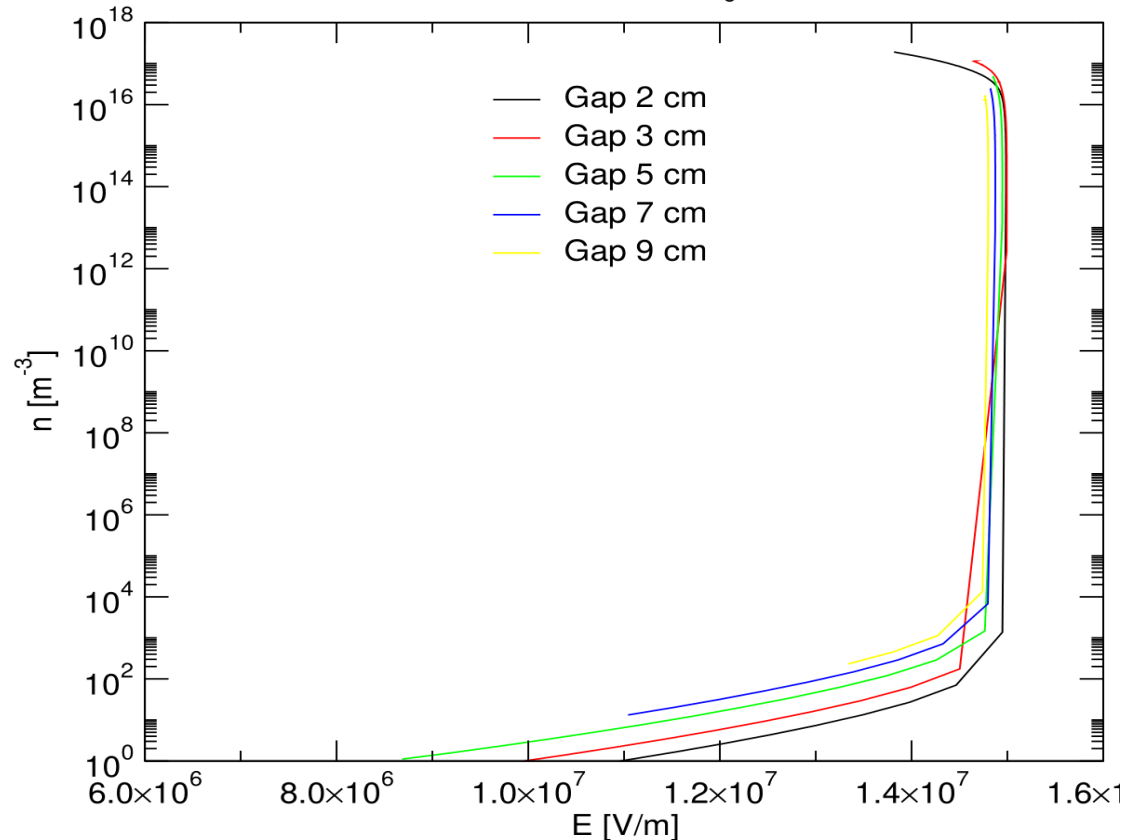
$$\hat{S}_k(r) = \int_{R_e}^r S_k r' dr' \quad k = N_2^+, O_2^+$$

G. Calìò, F. Ragazzi, A. Popoli, A. Cristofolini, L. Valdetaro, C. de Falco, and P. Barbante. Hierarchical multiscale modeling of positive corona discharges. J. Electrostatics, 141:104291, 2026



Number density positive ions vs emitter electric field

With photoionization, $R_e = 50 \mu\text{m}$



Exploits 1D results to fit an injection law, in the form of a Dirichlet condition, for the 2D model of the drift region

Kaptzov Hypothesis

The electric field on the emitting electrode increases proportionally to the voltage until a certain value, called the corona onset level. After that the electrical field at the injecting electrode remains unchanged and equal to the onset value.

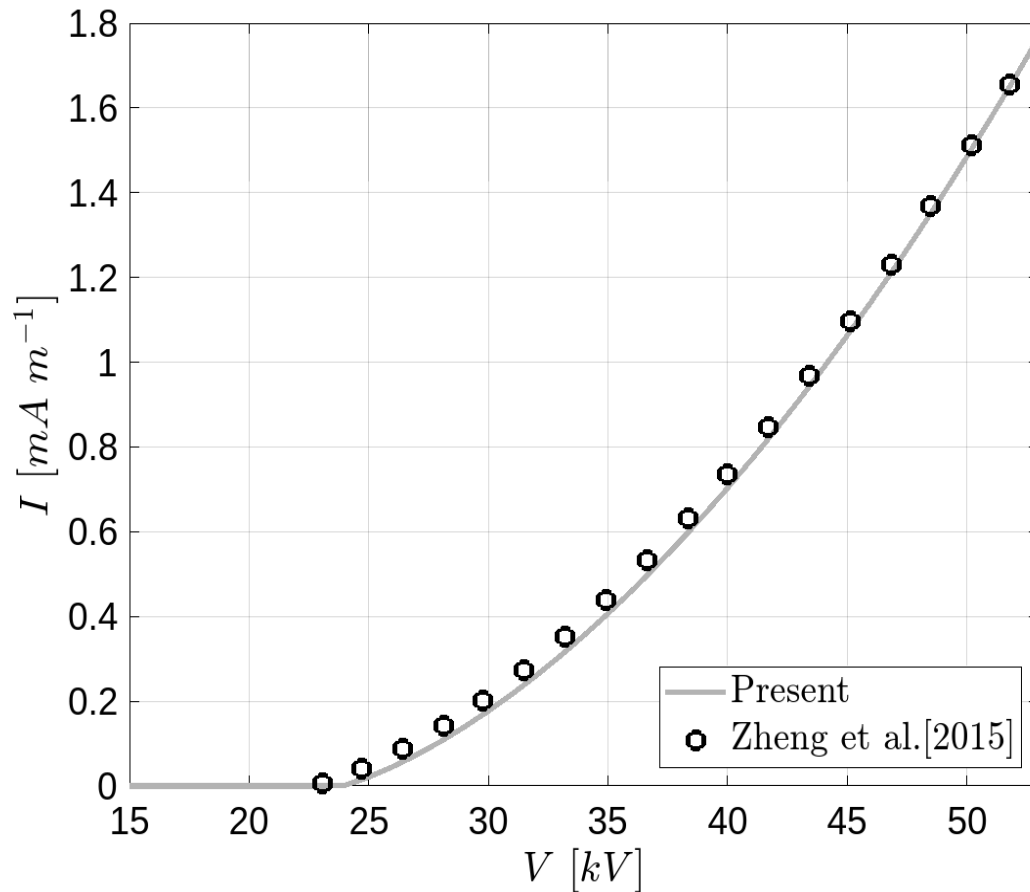
To enforce approximately this behaviour by means of a local condition we let:

$$n = n_{ref} e^{\frac{E \cdot \hat{n} - E_{ON}}{E_{ref}}} \text{ on } \partial\Omega_e$$

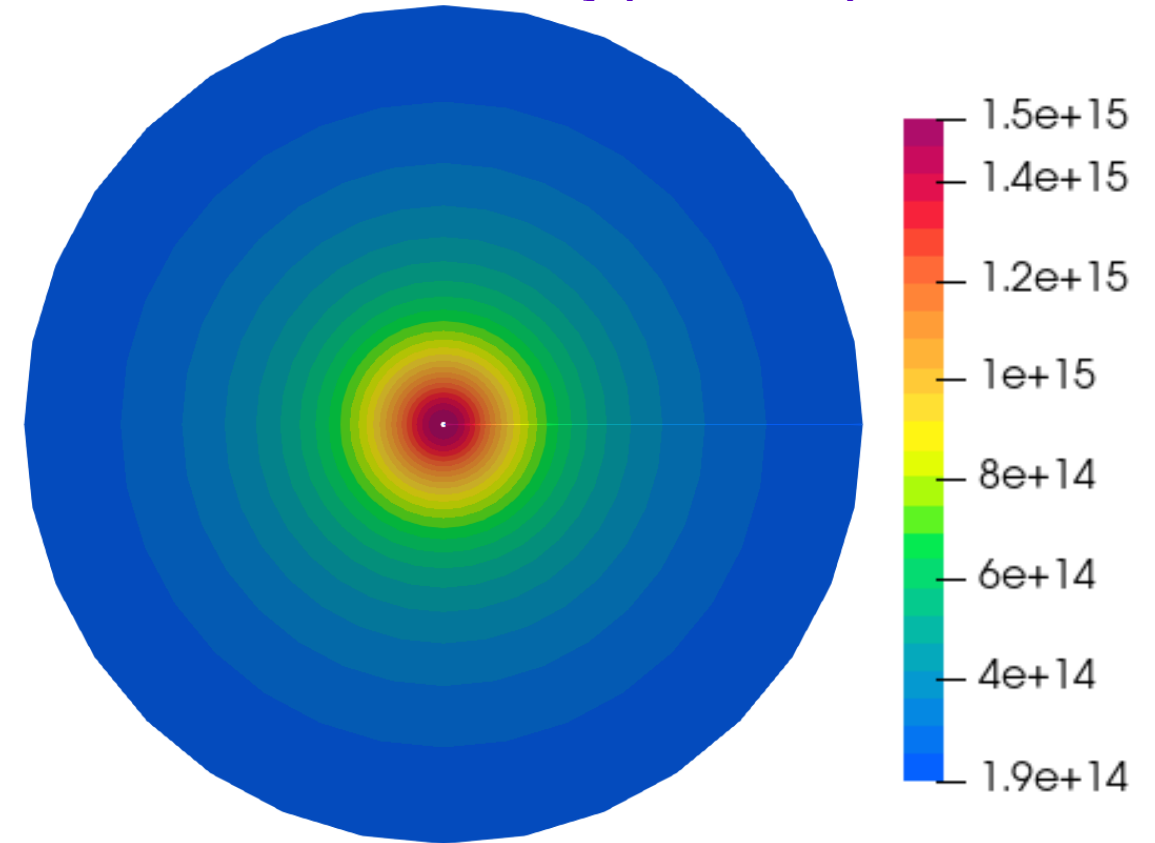
Corona inception when $|E| > E_{ON}$

Results: Macro-Scale Model

Current-Voltage Characteristic

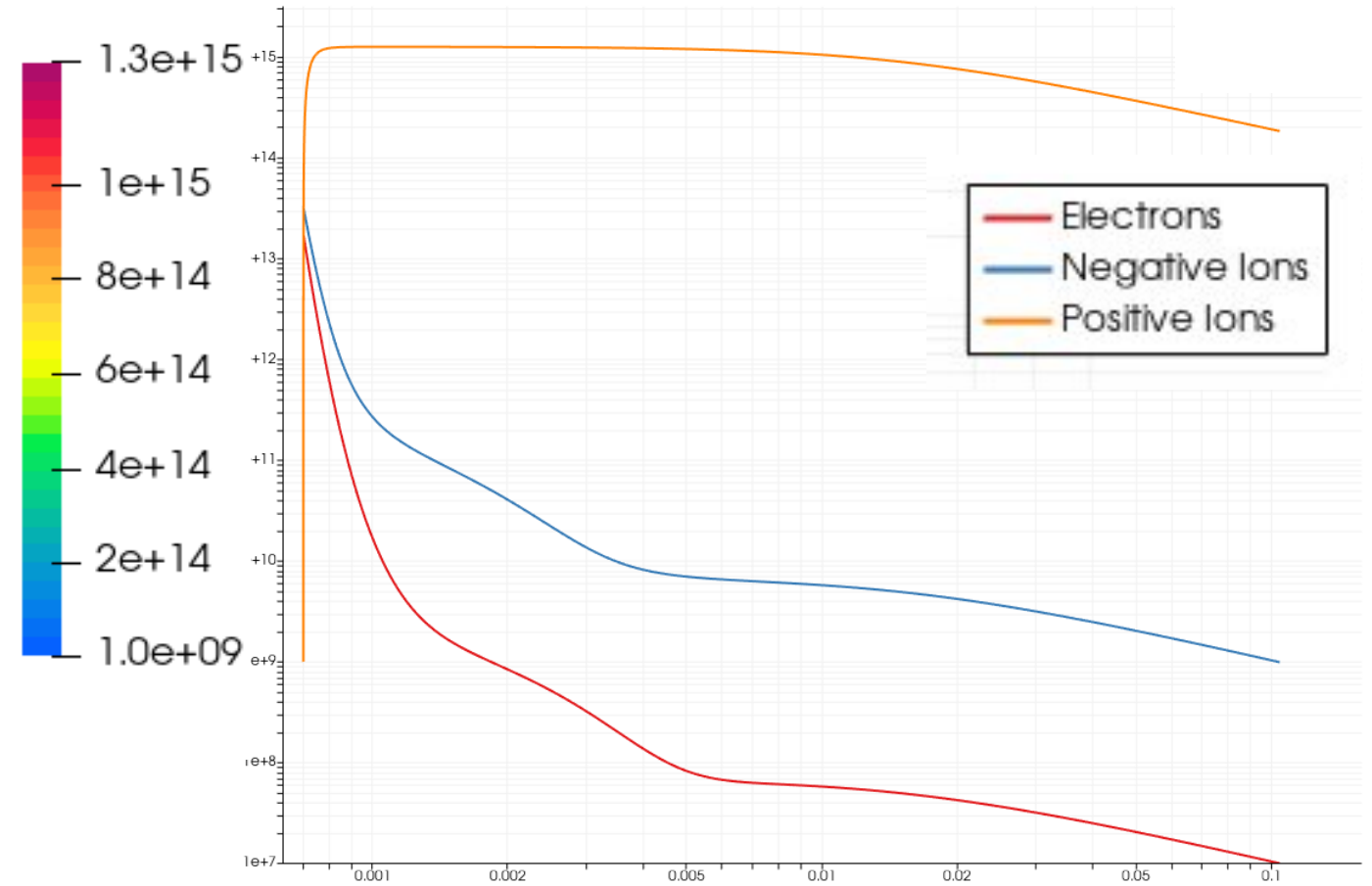
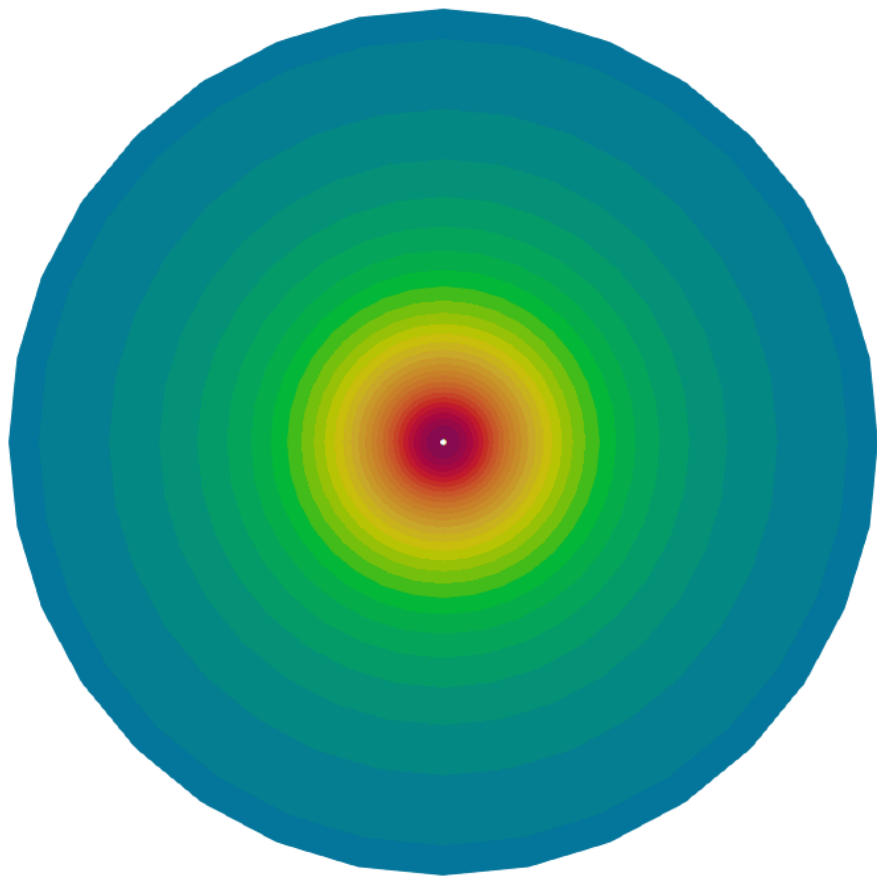


Number density (Va=50 kV)



Wire-in-Cylinder configuration: gap $L = 10,35 \text{ cm}$, emitter $R_e = 70$

Results: 3-species Model



3

Numerical Methods



Numerical Methods

Full Scale Model

1. **Spatial discretization Finite Volume** method with Scharfetter-Gummel stabilized fluxes in cylindrical coordinates
2. **Variable Order BDF** method for **time stepping**.
3. **Slowly varying applied voltage** to recover **steady** solutions

Implementation in **MATLAB**.
IDA solver (part of **SUNDIALS**) library for time integrations

Macro and 3-species Models

1. **Spatial discretization CVFEM-SG**: Control Volume-Finite Element method with Scharfetter-Gummel representations of the edge currents
2. **Newton Method** with continuation approach for the **non-linearity**

Implementation in **deal.II** (c++ finite element library), parallelization with **MPI**.
 Mesh of **quadrilateral elements** (in 2D), or hexahedral elements (in 3D);
 With non-conforming hierarchical refinement

Sundials, <https://computing.llnl.gov/projects/sundials>

D. Arndt, W. Bangerth, M. Bergbauer, B. Blais, M. Fehling, R. Gassmüller, T. Heister, L. Heltai, M. Kronbichler, M. Maier, P. Munch, S. Scheuerman, B. Turcksin, S. Uzunba-jakau, D. Wells, and M. Wichrowski. The deal.ii library, version 9.7. Journal of Numerical Mathematics, 33(4):403–415, 2025



Numerical Methods: CVFEM-SG

Why CVFEM-SG?

- Local current conservation
- Stable and accurate on unstructured grids
- Stable in strong-drift regimes

CVFEM-SG method by Bochev et al. *A new control volume finite element method for the stable and accurate solution of the drift-diffusion equations on general unstructured grids.*



Numerical Methods: CVFEM-SG

Consider the reference problem:

$$\begin{aligned}\nabla \cdot \mathbf{J} &= 0 \quad \text{on } \Omega \\ \mathbf{J} &= -D\nabla n - \mu\nabla\varphi n\end{aligned}$$

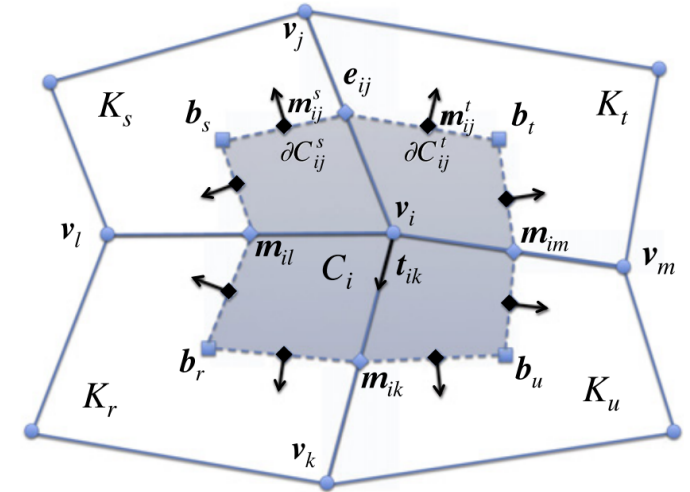
Integration on control volume C_i corresponding to the vertices in the mesh and the use of the Divergence Theorem lead

$$\int_{\partial C_i} \mathbf{J} \cdot \hat{\mathbf{n}} dS = 0 \quad \xrightarrow{\text{Discretization}} \quad \int_{\partial C_i} \mathbf{J}(n_h) \cdot \hat{\mathbf{n}} dS = 0$$

Base CVFEM

$$n(\mathbf{x}, t) \approx n_h(\mathbf{x}, t) = \sum_{v_j \in V} n_j(t) N_j(\mathbf{x})$$

$$\mathbf{J}(n_h) = -D\nabla n_h - \mu\nabla\varphi n_h = \sum_{v_j \in V} n_j(t) (-D\nabla N_j - \mu\nabla\varphi N_j) = \sum_{v_j \in V} n_j(t) \mathbf{J}(N_j)$$



Nodal finite element current density is unable to represent accurately the carrier transport between neighbouring nodes





Numerical Methods: CVFEM-SG

Replace nodal finite element current density:

$$\mathbf{J}_E = \sum_{e_{ij} \in E(\Omega)} h_{ij} J_{ij} \vec{W}_{ij} \quad \mathbf{J}_E \in \mathbb{H}(\text{curl}, \Omega)$$

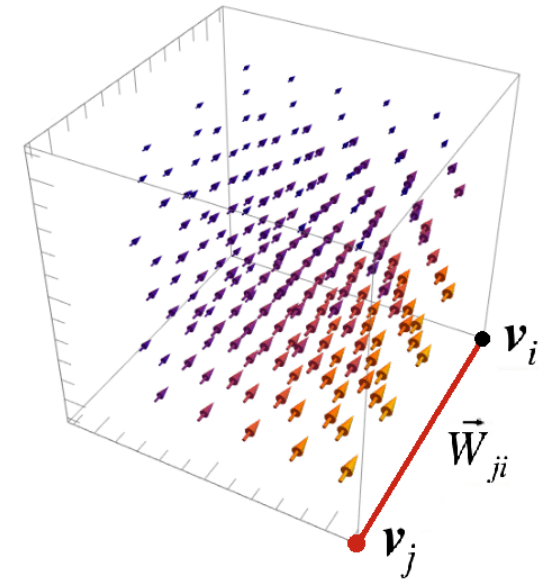
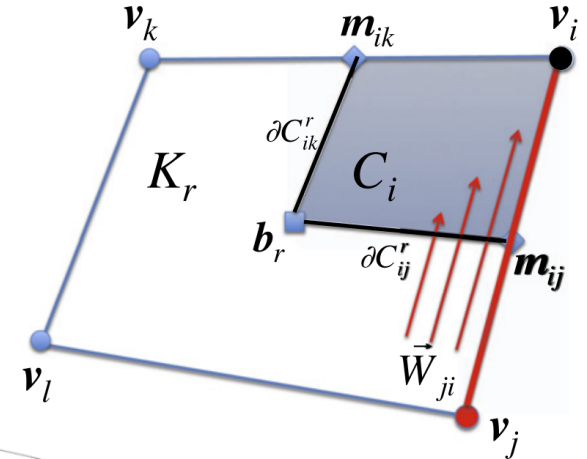
$$\mathbb{H}(\text{curl}, \Omega) = \{v \in L^2 : \nabla \times v \in L^2\}$$

First Order **Nédélec basis functions** of the first kind

1D Scharfetter–Gummel formula

$$J_{ij} = \frac{D}{h_{ij}} [n_j B(-2a_{ij}) - n_i B(2a_{ij})] \quad a_{ij} = -\frac{\mu(\varphi_j - \varphi_i)}{2D}$$

CVFEM-SG method by Bochev et al. A new control volume finite element method for the stable and accurate solution of the drift-diffusion equations on general unstructured grids.





Numerical Methods: CVFEM-SG

1D Scharfetter–Gummel formula

$$J_{ij} = \frac{D}{h_{ij}} [n_j B(-2a_{ij}) - n_i B(2a_{ij})] \quad a_{ij} = -\frac{\mu(\varphi_j - \varphi_i)}{2D}$$

Bernoulli Function

$$B(x) = \frac{x}{\exp(x) - 1}$$

It comes from the solution of the following 1D problem on each edge:

$$\frac{\partial J_{ij}}{\partial s} = 0; \quad J_{ij} = -E_{ij}n(s) + D \frac{\partial n(s)}{\partial s}$$

$$n(0) = n_i \quad \text{and} \quad n(h_{ij}) = n_j$$

Electric Field

$$E_{ij} = -\frac{(\varphi_j - \varphi_i)}{h_{ij}}$$

CVFEM-SG method by Bochev et al. *A new control volume finite element method for the stable and accurate solution of the drift-diffusion equations on general unstructured grids.*





Numerical Methods: CVFEM-SG

$$\int_{\partial C_i} \mathbf{J} \cdot \hat{\mathbf{n}} dS = 0 \quad \xrightarrow{\text{Discretization}} \quad \int_{\partial C_i} \mathbf{J}_E \cdot \hat{\mathbf{n}} dS = 0$$

$$\mathbf{J}_E = \sum_{v_j \in V} \sum_{e_{jk} \in E(v_j)} D n_j B(2a_{jk}) \vec{W}_{jk}$$

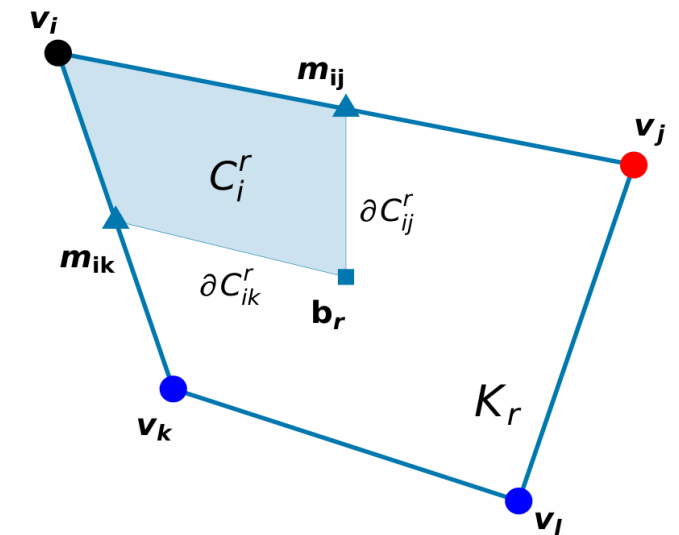
$$\mathbf{J}_E \in \mathbb{H}(\text{curl}, \Omega)$$

$$\mathbb{H}(\text{curl}, \Omega) = \{v \in L^2 : \nabla \times v \in L^2\}$$

Local Matrix Contribution on a cell K^r

$$\begin{aligned} \mathbf{K}_{ij}^r = & DB(2a_{jl}) \left(\int_{\partial C_{ij}^r} \vec{W}_{jl} \cdot \hat{\mathbf{n}} dS + \int_{\partial C_{ik}^r} \vec{W}_{jl} \cdot \hat{\mathbf{n}} dS \right) \\ & + DB(2a_{ji}) \left(\int_{\partial C_{ij}^r} \vec{W}_{ji} \cdot \hat{\mathbf{n}} dS + \int_{\partial C_{ik}^r} \vec{W}_{ji} \cdot \hat{\mathbf{n}} dS \right) \end{aligned}$$

CVFEM-SG method by Bochev et al. A new control volume finite element method for the stable and accurate solution of the drift-diffusion equations on general unstructured grids.





Numerical Methods: CVFEM-SG

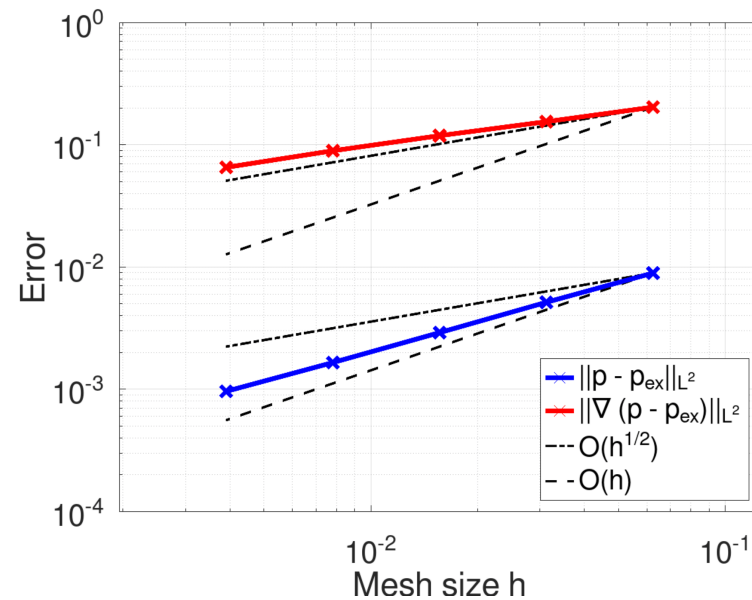
$$\begin{cases} \nabla \cdot (-D \nabla n_p - \mu \nabla \varphi n_p) = f, & \text{in } \Omega \subset \mathbb{R}^n, \\ n_p = g, & \text{on } \partial\Omega. \end{cases}$$

$$\Omega = (0, 1)^n$$

$$D = 10^{-5}$$

f, g are such that $n_{p,\text{ex}}$ is imposed

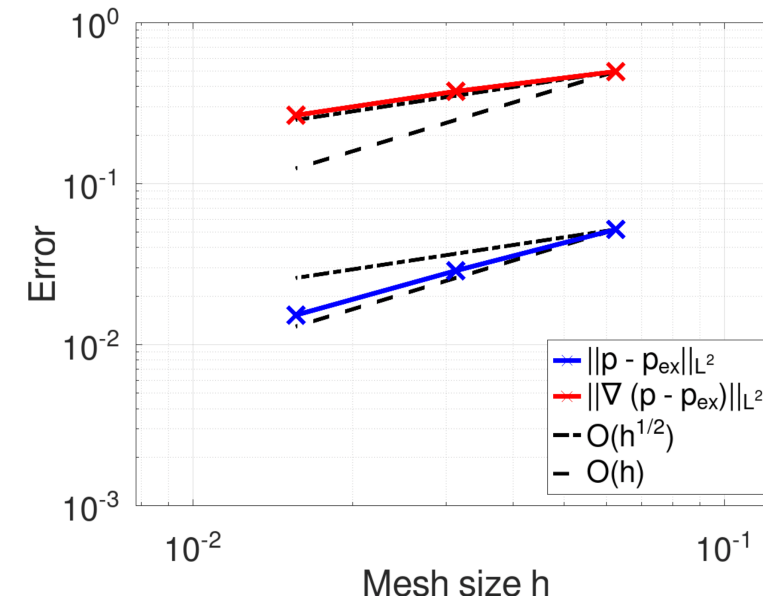
2D case error plot on a randomly perturbed mesh



$$-\mu \nabla \varphi = \begin{bmatrix} -\sin(\pi/6) \\ \cos(\pi/6) \end{bmatrix}$$

$$n_{p,\text{ex}} = x^3 - y^2$$

3D case error plot on a randomly perturbed mesh



$$-\mu \nabla \varphi = \begin{bmatrix} -\sin(\pi/6) \\ \cos(\pi/6) \\ 1 \end{bmatrix}$$

$$n_{p,\text{ex}} = x^3 - y^2 + z^4$$



Numerical Methods: CVFEM-SG

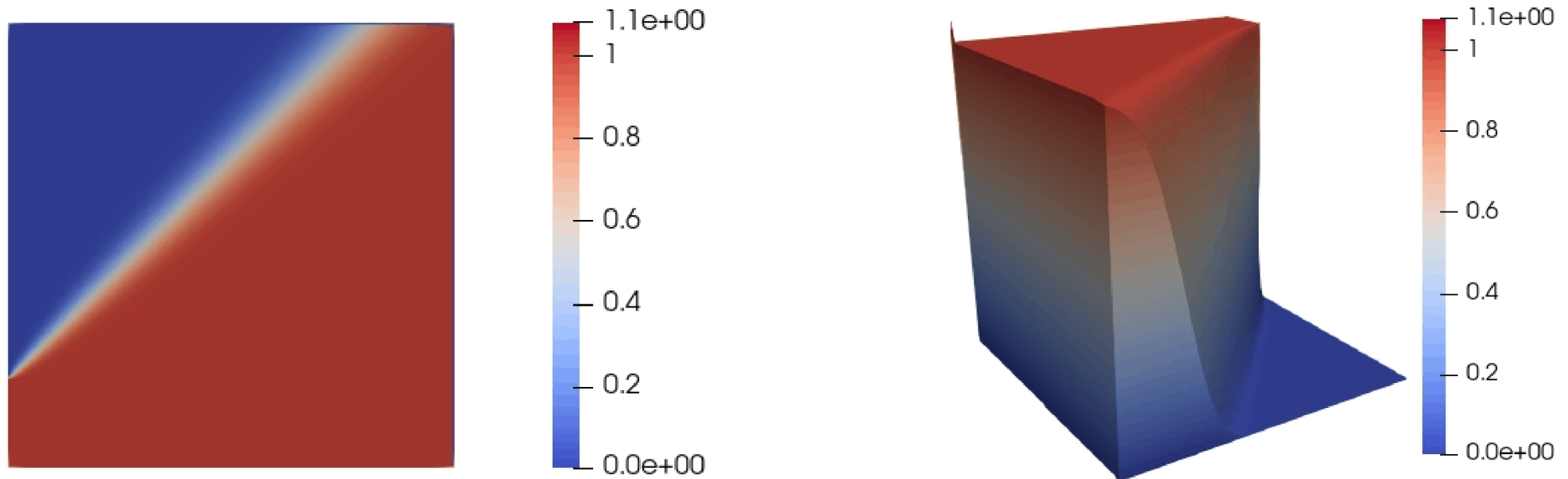
$$\begin{cases} \nabla \cdot (-D \nabla n_p - \mu \nabla \varphi n_p) = f, & \text{in } \Omega \subset \mathbb{R}^n, \\ n_p = g, & \text{on } \partial\Omega. \end{cases}$$

$$g = \begin{cases} 1 & \text{on } \{0 \leq x \leq 1, y = 0\} \cup \{x = 0, 0 \leq y \leq 0.2\} \\ 0 & \text{otherwise.} \end{cases}$$

$$\Omega = (0, 1)^n$$

$$D = 10^{-5}$$

$$f = 0$$



4

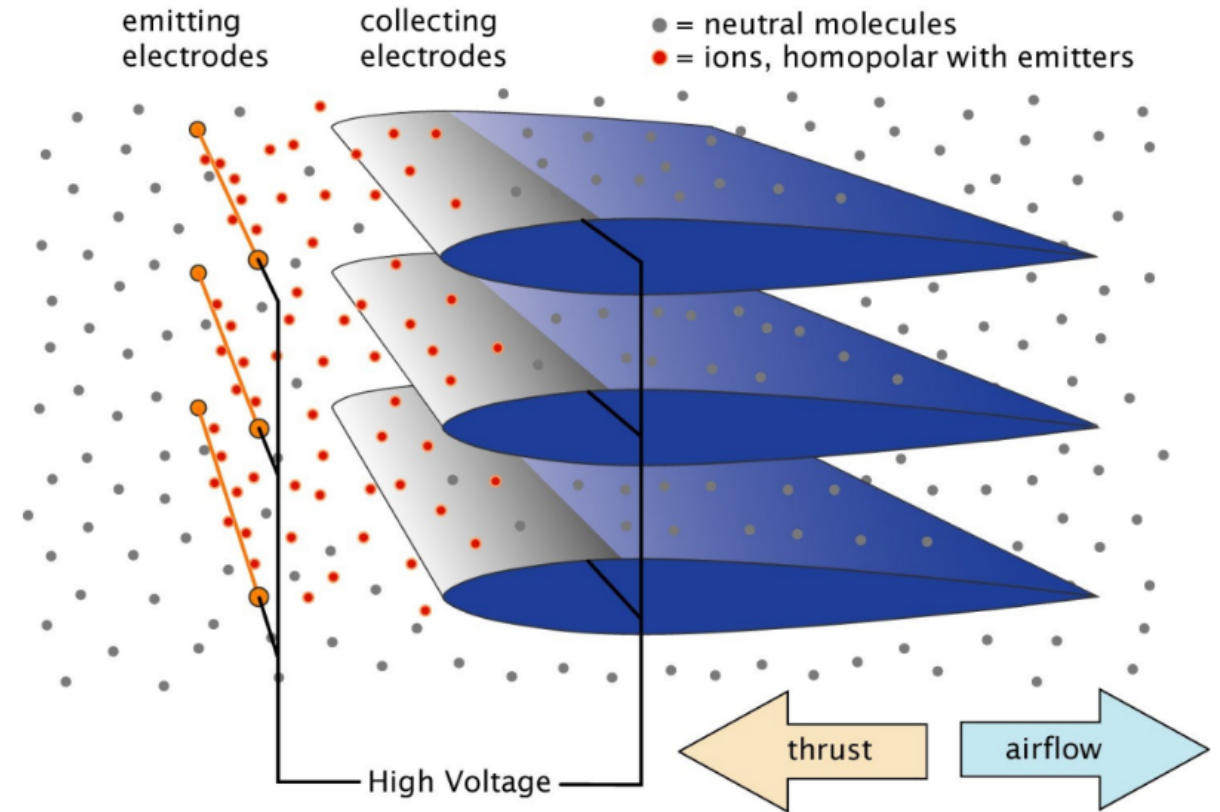
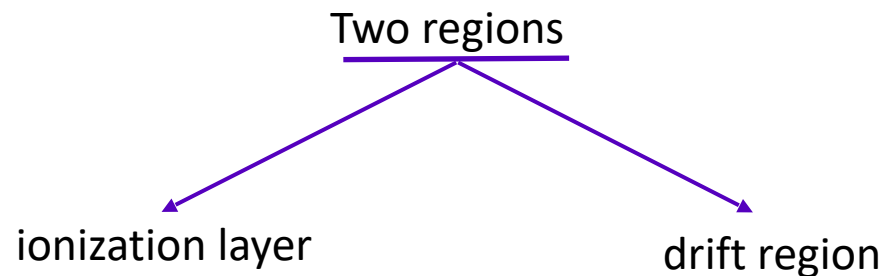
Ionic Propulsion in Atmosphere

Principles of Ionic Propulsion in Atmosphere

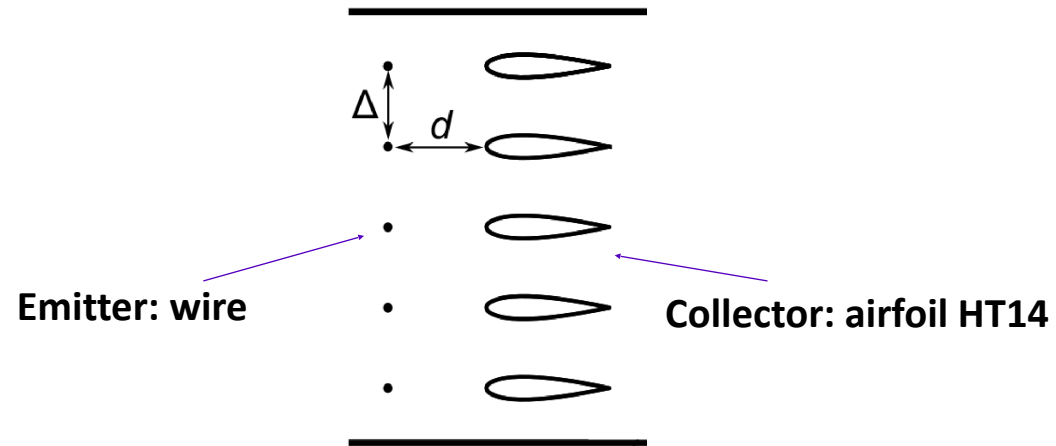
The most important components of a Ionic Thruster are the two electrodes:

- Emitter: high voltage electrode
- Collector: grounded.

The underlying physical phenomenon is known as **Positive Corona Discharge**.

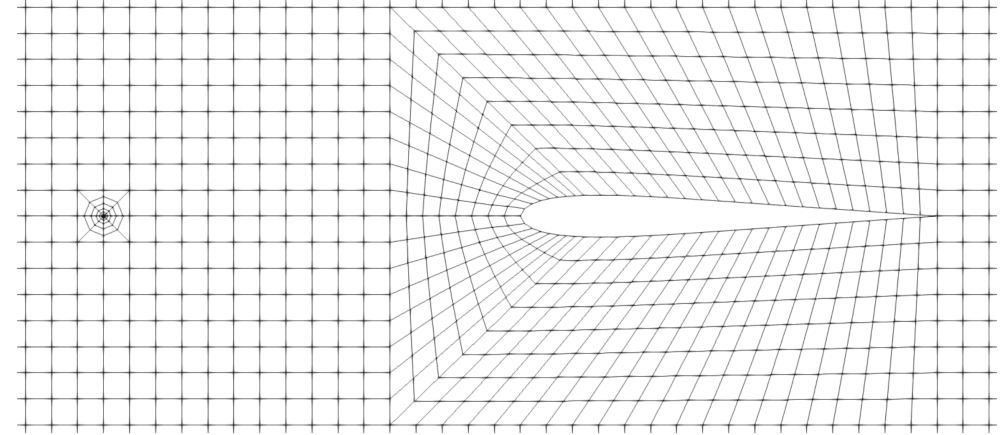


Wire – Airfoil: Geometry

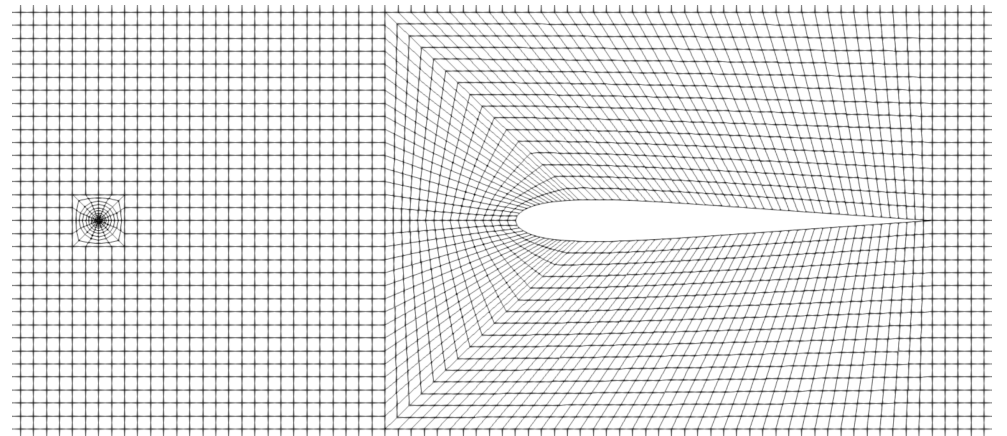


Variable	Value	Units
Radius emitter	28	μm
HT14 chord	20	mm
HT14 thickness	2	mm
Gap d	15, 20, 30	mm
Spacing Δ	13.8, 20, 90	mm

Section of coarse mesh

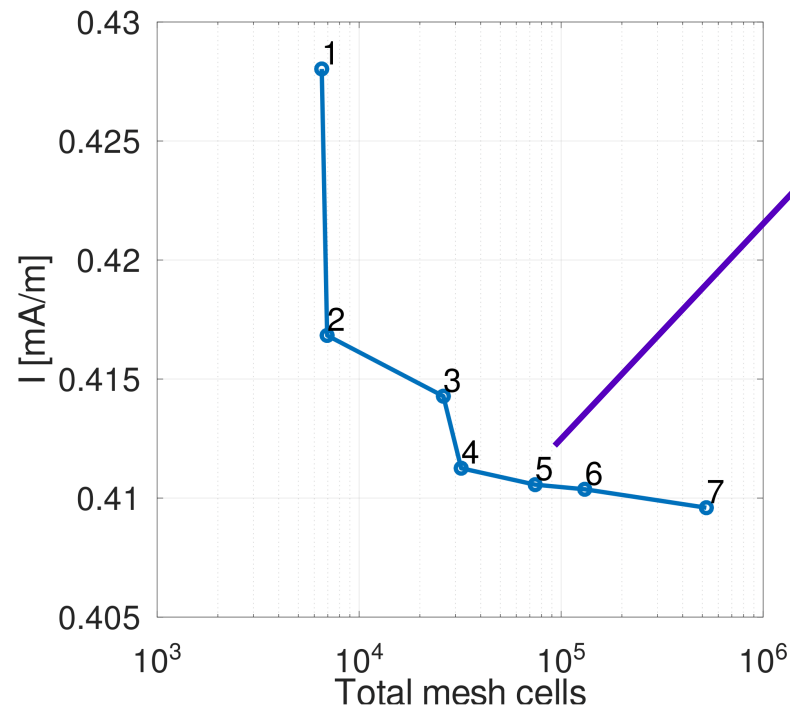


Section of mesh after 1 global refinement



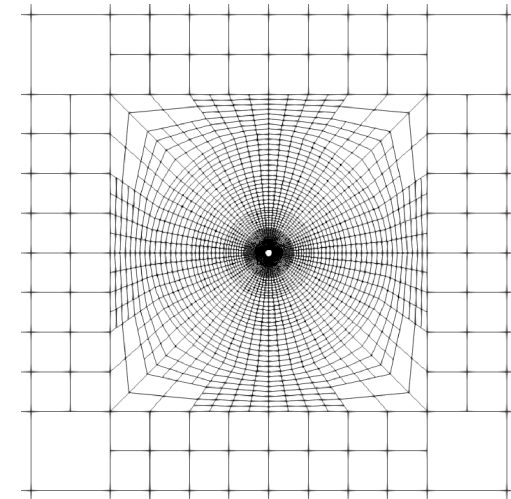
Wire – Airfoil: Grid Convergence

Total current as function of mesh refinements

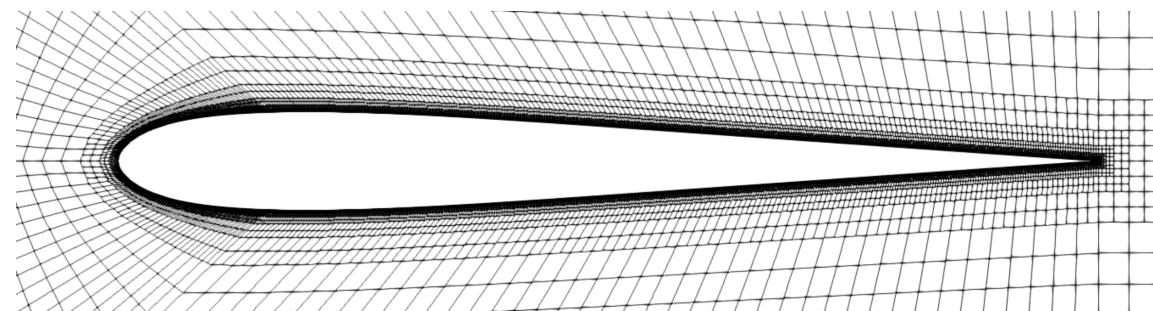


Mesh 5 example

Emitter mesh after 1 global and 2 local refinements

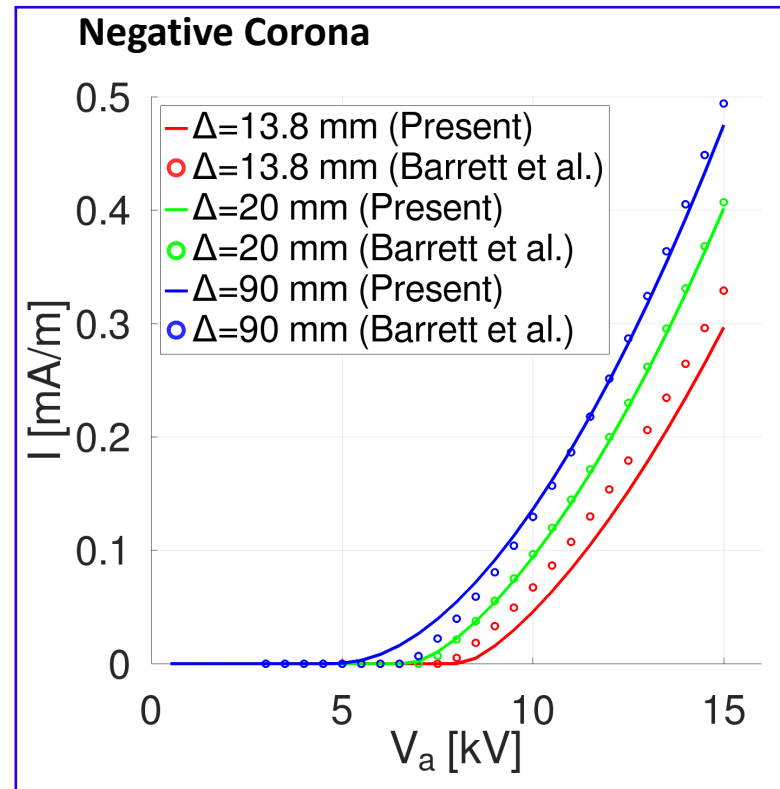
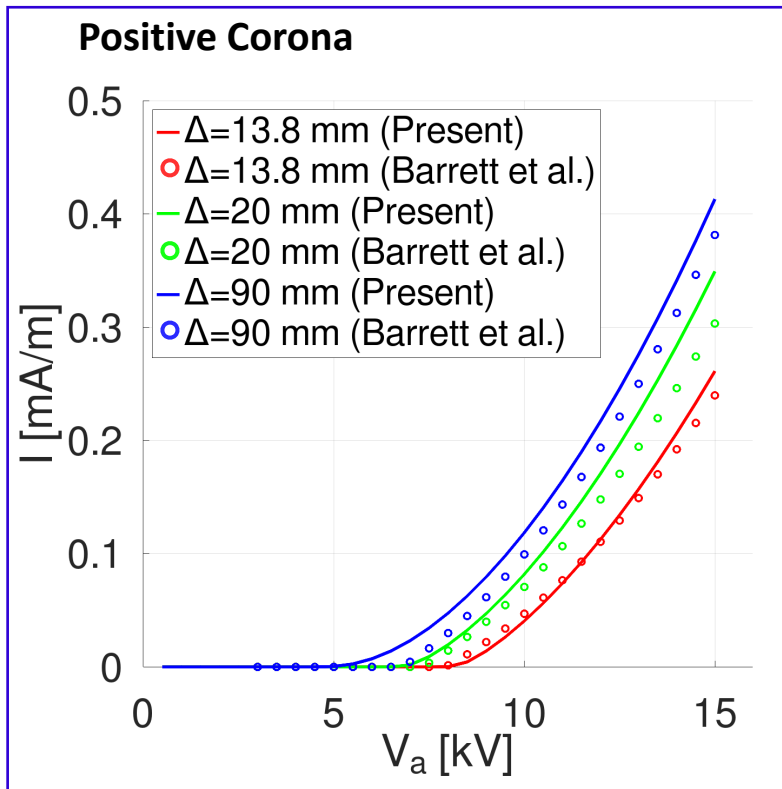


Collector mesh after 1 global and 6 local refinements



Wire – Airfoil: Validation

Current-Voltage characteristics at $d=20$ mm, comparison with experimental data by Barrett et al.

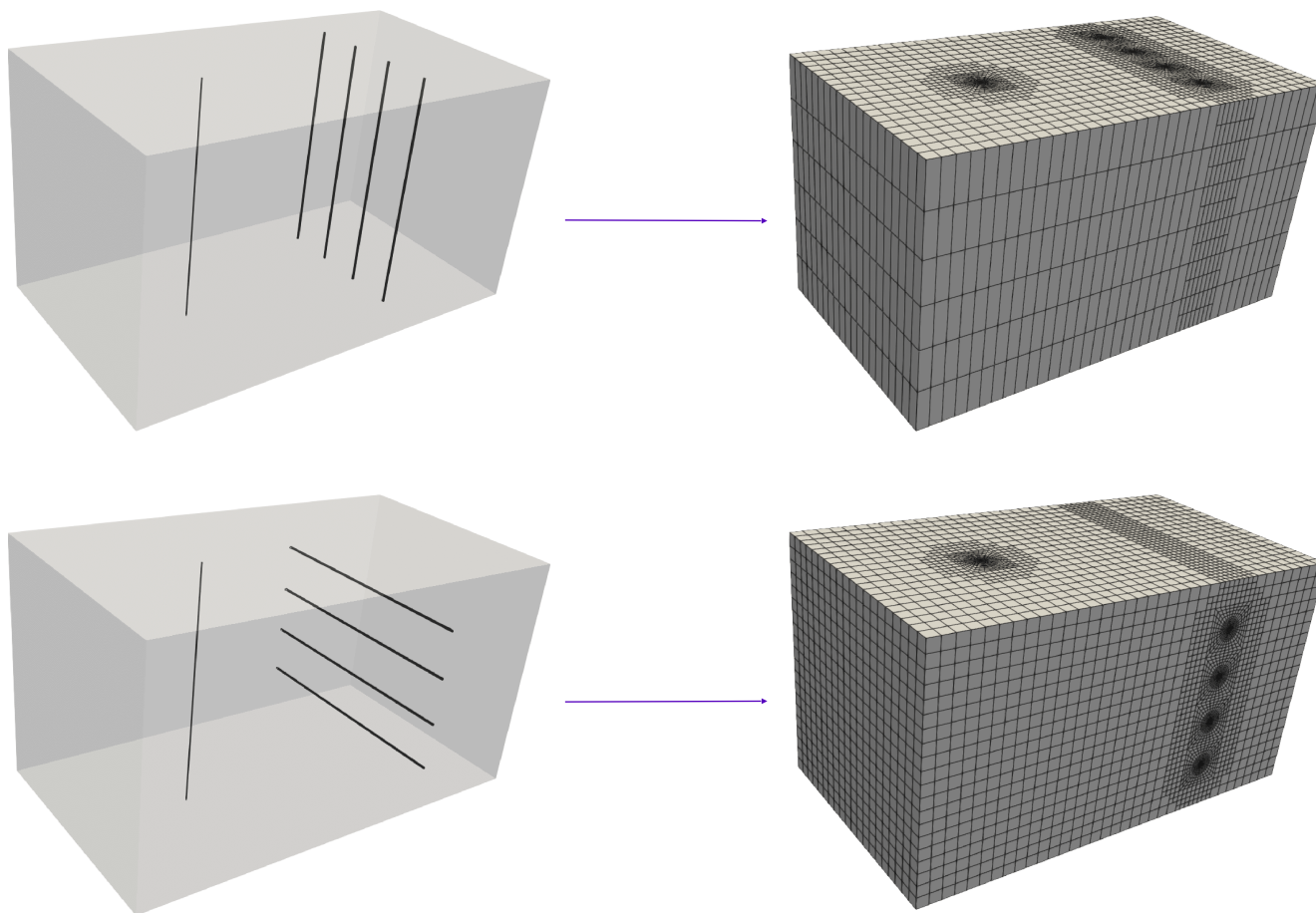


Main Findings:

- Errors from experimental data are within 10%.
- Good inception voltage predictions.
- Lower spacing Δ decreases the current.

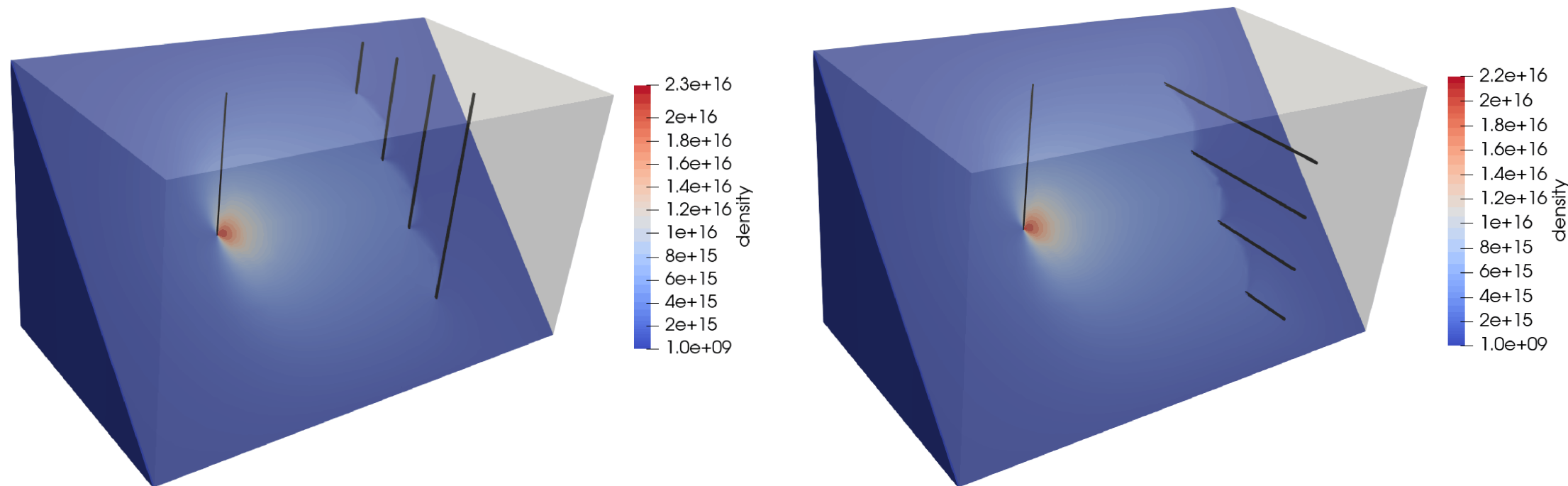
Experimental model of Battett et al. *Order-of-magnitude improvement in electroaerodynamic thrust density with multistaged ducted thrusters.*

Three Dimensional Configuration

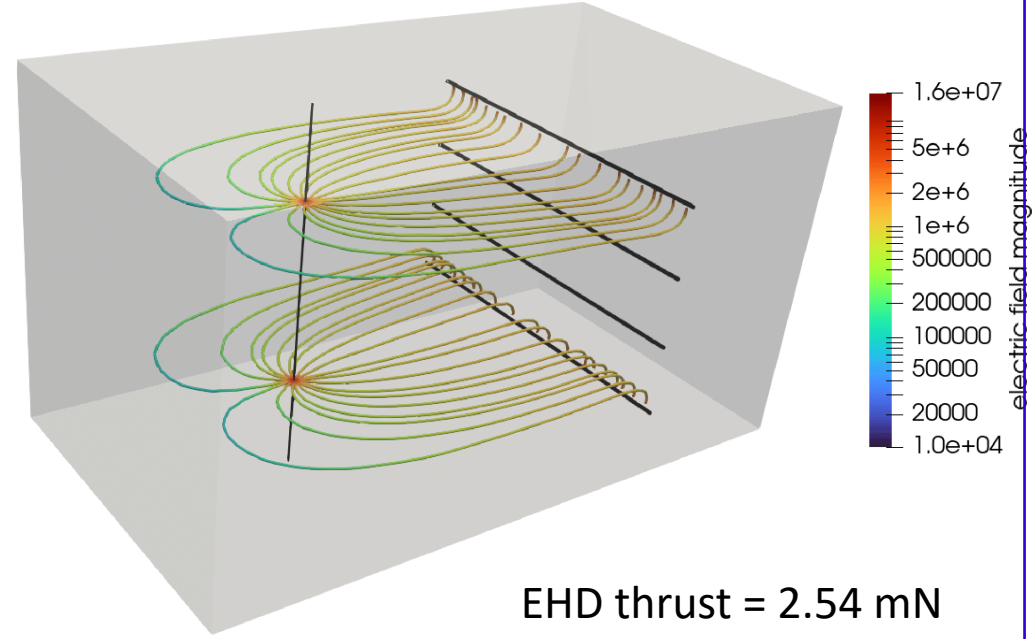
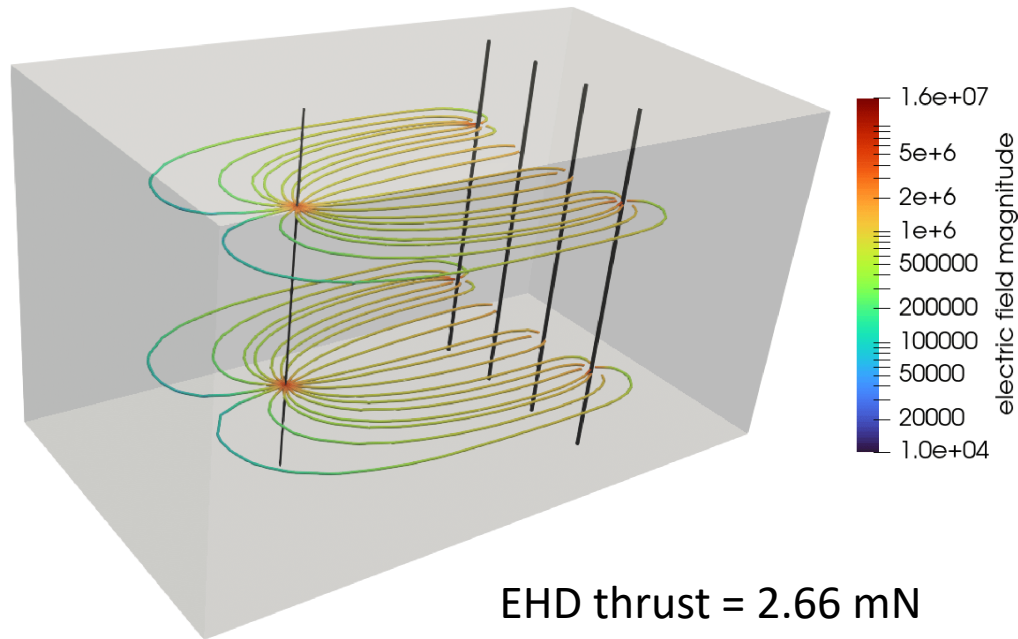


Variable	Value	Units
Radius emitter	50	μm
Radius collector	100	μm
Gap d	16	mm
Spacing Δ	4	mm
Wire length	20	mm

Number densities for both configurations



Some electric field lines for both configurations



$$T_{EHD} = \int_{\Omega} q n E dV$$

Notes:

- The first configuration can be simulated in 2D, it was used to validate the 3D implementation.
- The two configurations are almost equivalent in performance.

5

Coupling Corona with Fluid-Dynamics

Coupling with the Fluid Problem

The ionic wind forces the air to move. The airflow is modeled by the Incompressible Navier-Stokes equations:

Incompressible Navier-Stokes equations

$$\begin{aligned} \rho_f \frac{\partial \mathbf{u}}{\partial t} - \mu_f \Delta \mathbf{u} + \rho_f (\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla p &= \mathbf{f}_{EHD} \quad \text{in } \Omega, t > 0 \\ \nabla \cdot \mathbf{u} &= 0 \quad \text{in } \Omega, t > 0 \end{aligned}$$

where $\mathbf{f}_{EHD} = q(n_p - n_n) \mathbf{E}$

$\mathbf{u}$: average velocity of the mixture

p : average pressure of the mixture

$n_p, n_n, \mathbf{E}$: results of Corona problem

Boundary condition on inlet/outlet: *Regularized directional do-nothing boundary conditions for the navier-stokes equations.*

Boundary and Initial Conditions

$$\mathbf{u}(\mathbf{x}, 0) = \mathbf{0} \quad \text{in } \Omega, t = 0$$

no-slip on electrodes

$$\mathbf{u} = \mathbf{0} \quad \text{on } (\Gamma_e \cup \Gamma_c), t > 0$$

regularized directional do-nothing on inlet and outlet

$$\tilde{\mathbb{T}}^f \hat{\mathbf{n}} + \rho_f \frac{1}{2} (\mathbf{u} \cdot \hat{\mathbf{n}})_\delta^- \mathbf{u} = \mathbf{0} \quad \text{on } (\Gamma_{in} \cup \Gamma_{out}), t > 0$$

free-slip on upper and lower boundary

$$\mathbf{u} \cdot \hat{\mathbf{n}} = 0; \quad (\tilde{\mathbb{T}}^f \hat{\mathbf{n}}) \cdot \hat{\boldsymbol{\tau}} = 0 \quad \text{on } (\Gamma_{bot} \cup \Gamma_{up}), t > 0$$

$$\text{where} \quad \tilde{\mathbb{T}}_{ij}^f = \mu_f \frac{\partial u_i}{\partial x_j} - p \delta_{ij}$$

Thrust Computation

Method 1

It can be computed by performing an integral balance of the momentum on a control volume containing the electrodes:

$$T = \int_{\partial\Omega_{out}} p \mathbf{n} ds + \int_{\partial\Omega_{in}} p \mathbf{n} ds + \int_{\partial\Omega_{out}} \rho_f \mathbf{u} (\mathbf{u} \cdot \mathbf{n}) ds + \int_{\partial\Omega_{in}} \rho_f \mathbf{u} (\mathbf{u} \cdot \mathbf{n}) ds$$

The contribution of the upper and lower boundary is zero because of the free-slip condition, and the viscous term on the lateral boundaries has been neglected.

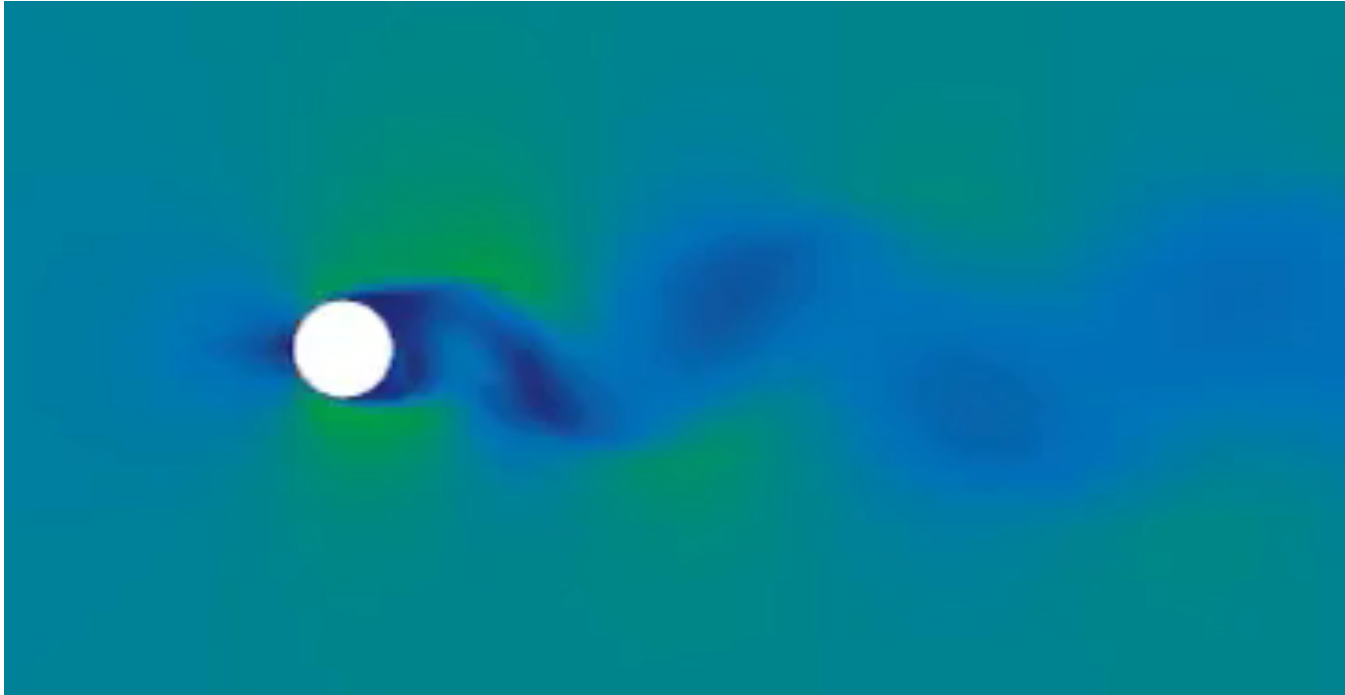
Method 2

It can be computed by considering the forces acting on the electrodes:

$$T_x = D + T_{EHD}$$

$$T_{EHD} = \int_{\Omega} \mathbf{f}_{EHD} dV \quad D = \int_{\Gamma_{electrodes}} \left(-p \mathbf{n} + \mu \left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right] \cdot \mathbf{n} \right) ds$$

Wire-to-Cylinder



gap $D = 30 \text{ mm}$

collector radius $R_c = 15 \text{ mm}$

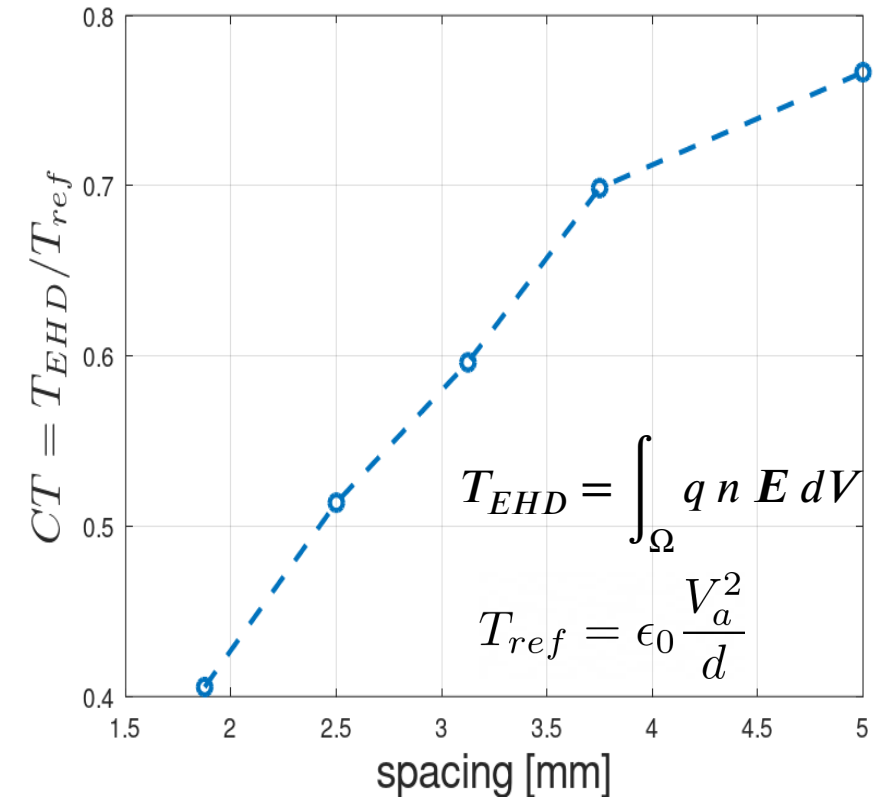
$V_a = 10 \text{ kV}$

Reference Thrust (mN/m)	Computed T
5,42	5,407

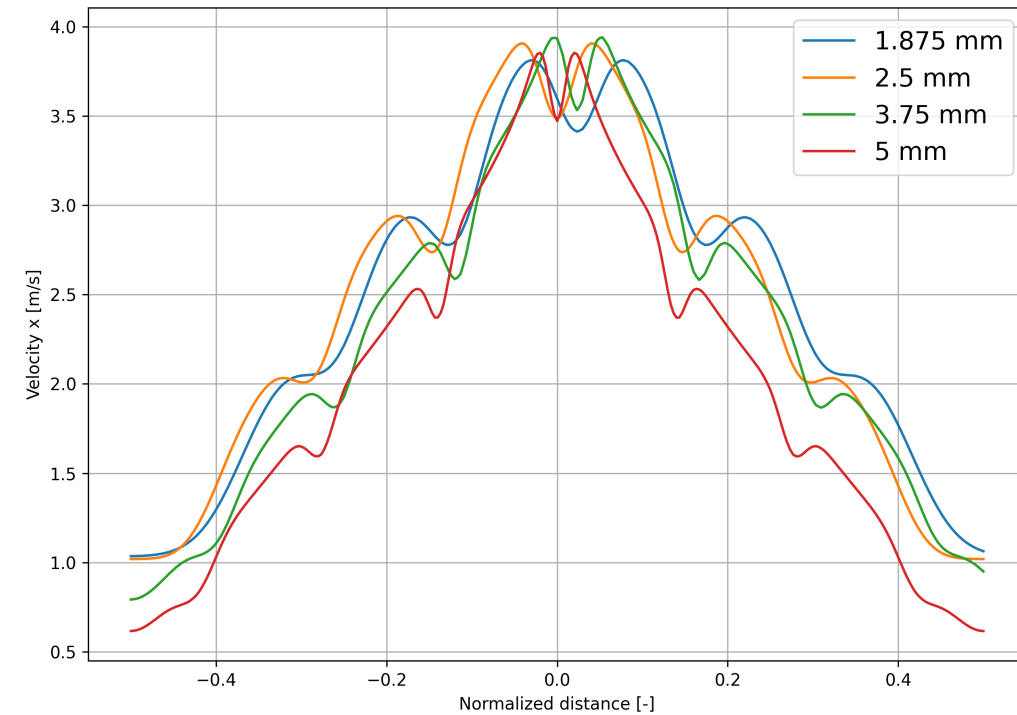
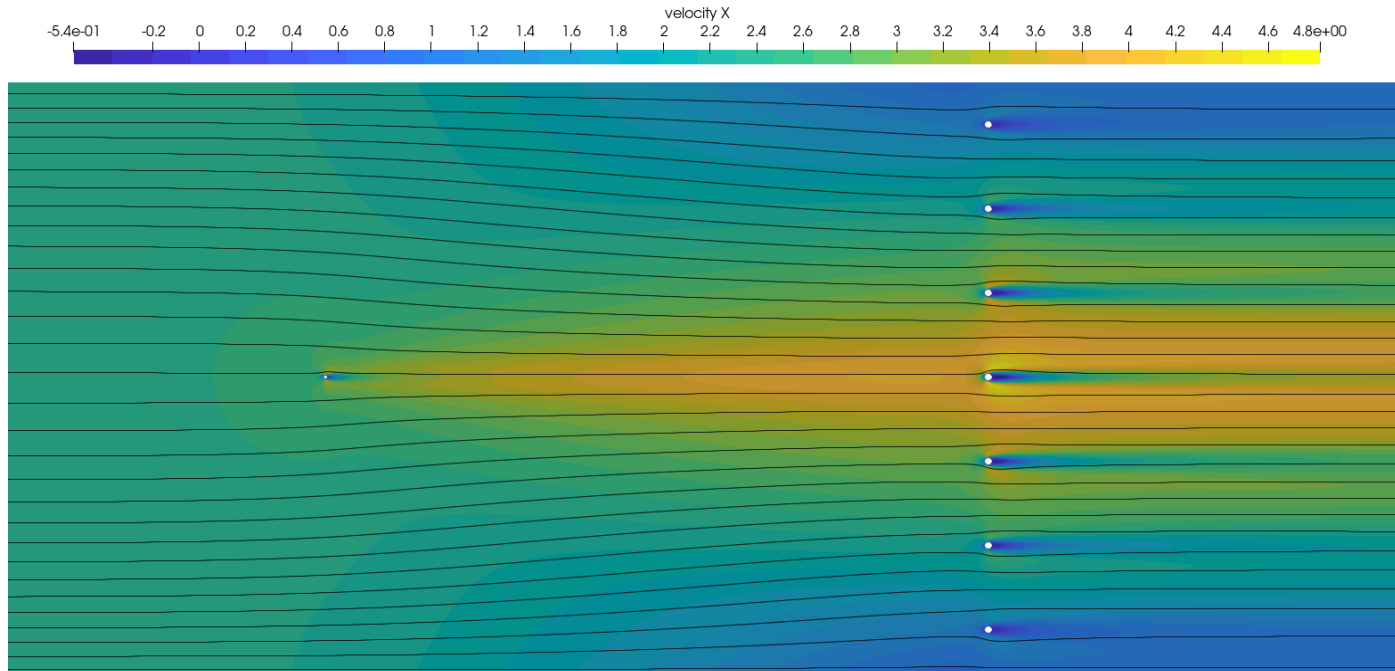
Single Stage



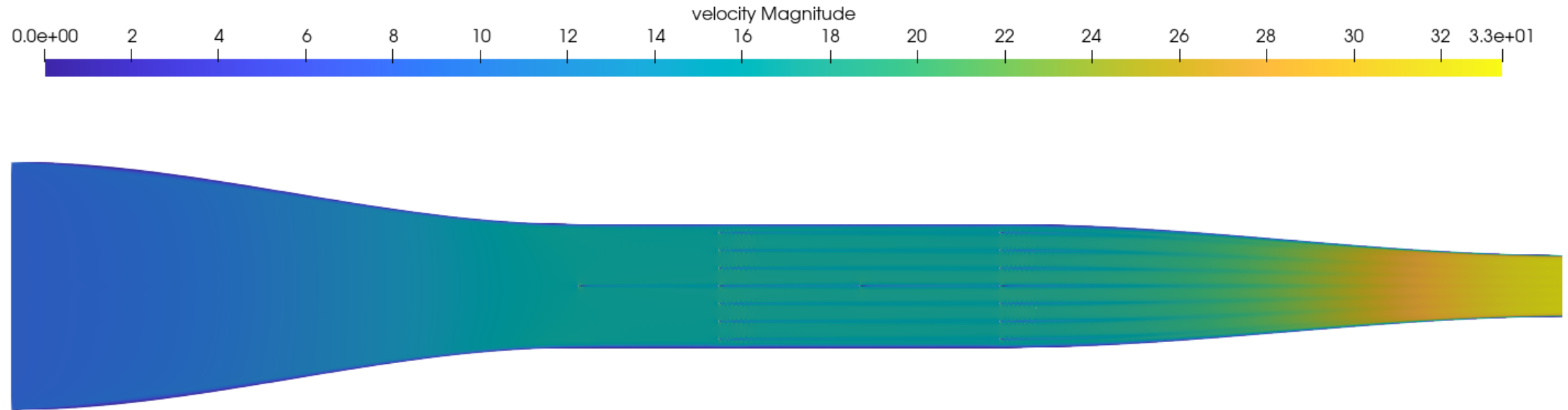
$$D = 20 \text{ mm} \quad R_c = 100 \text{ } \mu\text{m} \quad R_e = 100 \text{ } \mu\text{m} \quad V_a = 5 \text{ kV} - 20 \text{ kV}$$



Single Stage

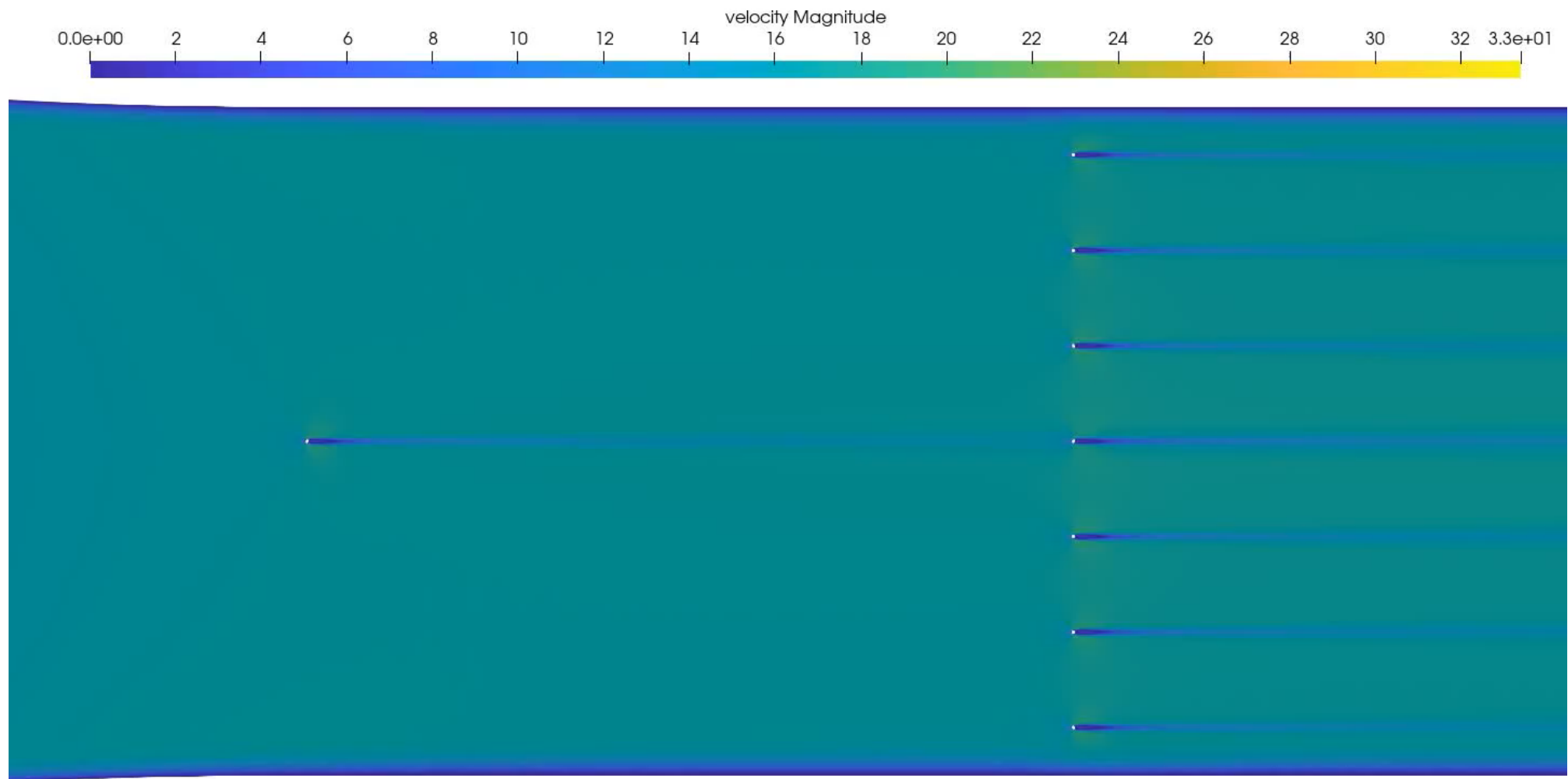


Multi-Stage Ducted Thruster



$$D = 20 \text{ mm} \quad R_c = 100 \text{ } \mu\text{m} \quad R_e = 100 \text{ } \mu\text{m} \quad V_a = 20 \text{ kV}$$

Inter-stage distance $s=20 \text{ mm}$



Thank You!